

Problem Set - 1994 (April 1994)

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The roots of the quadratic equation $2x^2 - 4x + 1 = 0$ are the radii of two circles. Without finding the roots, find the sum of the circumferences and the sum of the areas of the circles.
2. The sides of the rectangle $KLMN$ satisfy $KL : LM = 3 : 2$, and F is the midpoint of the side LM . A line is perpendicular to the segment KF and bisects the area of $KLMN$. In what ratio does this line divide the sides KL and MN ?
3. Find the real numbers x for which
$$\frac{\sqrt{x+2}-1}{\sqrt{x+3}-2} : \frac{\sqrt{x+4}-3}{\sqrt{x+13}-4} \geq 0$$
4. A right circular cone is cut by planes parallel to the base into a cone and two frustums. Their volumes are in the ratio $1 : 2 : 3$, starting from the apex. What is the ratio of the heights of these solids?
5. The equation of the axis of symmetry of the isosceles triangle PQR is $2y - x - 6 = 0$, and the points $P(2, 4)$ and $Q(-2, -1)$ are given. Find the length of the altitude from the missing vertex.
6. Three sides of an isosceles trapezoid have the same length. Find its angles so that its area is as large as possible.
7. Solve the following system of equations over the integers.

$$ab + c = 11$$

$$bc + a = 13$$

$$ca + b = 17$$

8. Find the smallest value of the expression $\sin^8 x + \cos^8 x$.