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translated from: KoMaL, February 1994, pages 63-64

Pdf file: KoMaL, February 1994, pages 63-64

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following system of equations over the pairs of real numbers.

$$\begin{aligned}x^2 - 2xy + y^2 &= 9 \\ 4x^2 + xy + 4y^2 &= 18\end{aligned}$$

2. In triangle ABC , $AB = 21$, $AC = 28$, $BC = 20$ units. The angle bisector from A meets the side BC at D , and the angle bisector from C meets the side AB at E . What fraction of the area of triangle ABC is the area of triangle AED ?
3. Take the sum of the first n positive integers that leave remainder 1 when divided by 3, and divide it by the sum of the first n positive integers that leave remainder 3 when divided by 4. What is n , if the quotient is $\frac{5}{7}$?
4. For which real numbers x is the value of the expression

$$\frac{2x^2 - 6x + 6}{x^2 - 4x + 5}$$

at least 1 and at most 3?

5. A convex octagon is inscribed in a circle of radius r , so that four of its sides are 1 unit long, and the other four sides are 3 units long. Find the radius of the circle.
6. Find the equation of the circle that is tangent to the coordinate axes and also to the line $3x + 4y = 10$.
7. For which values of the real parameter m does the equation

$$\frac{m}{12 \sin^4 x - 12 \sin^2 x + m} = 3$$

have a real solution? Solve the equation for $m = \frac{9}{2}$.

8. Prove that one root of the quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) is three times the other root if and only if the discriminant of the equation is $D = \frac{b^2}{4}$.