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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equation over the real numbers.

$$\frac{2}{x^2 - 9} = \frac{1}{x^2 - 3x} - \frac{x - 6}{x^2 + 3x}$$

2. The shorter side, the longer side, and the diagonal of a rectangle are, in this order, consecutive terms of a geometric sequence. What is the ratio of the geometric sequence? Find the angles between the diagonals and the sides.
3. Solve the following equations over the real numbers.

$$\text{a) } \sqrt{(\sqrt{x})^{\log x^2}} = 100 \qquad \text{b) } 10^{2x+3} = 2^{x+6} \cdot 125^x$$

4. The sum of the squares of the second and fourth terms of an arithmetic sequence is twice the sum of the squares of the first and third terms. The sum of the first four terms of the sequence is 20. Find the first four terms.
5. Two adjacent vertices of the rectangle $ABCD$ are $A(2, 0)$ and $D(-1, 1)$, and we know that $3 \cdot AD = AB$. How long are the segments that the circle circumscribed around the rectangle cuts off from the x -axis and from the y -axis?
6. Two perpendicular chords of a circle of radius 1 intersect each other so that the segments on one chord are in the ratio 2 : 3, and on the other one in the ratio 3 : 8. Find the lengths of the four arcs into which the chords divide the circle.
7. We want to make a box in the shape of a rectangular box whose base has area 4 square units, and the sum of the lengths of its 12 edges is 40 units. How should we choose the dimensions of the box so that its surface area is as large as possible? What is this largest surface area?
8. Solve the following equation over the pairs of real numbers.

$$\log_2 \left(\cos^2(xy) + \frac{1}{\cos^2(xy)} \right) = \frac{2}{y^2 - 4y + 6}$$