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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. In triangle ABC , $AB = 10$, $BC = 9$ units, and $\angle CAB = 60^\circ$. Find the length of the side AC .
2. The equations $x^2 + 3px + q = 0$ and $x^2 + px - q = 0$, where p and q are real parameters, have a common real root. Prove that the other two roots either add up to zero, or one of them is three times the other.
3. Solve the following equation over the real numbers.

$$\cos 2x + \sqrt{\frac{1 - \sin 2x}{2}} = 0$$

4. The base of the isosceles triangle ABC is $BC = 27$ units, and the radius of its inscribed circle is $\varrho = 9$ units. Find the radius of the circle that is tangent to the inscribed circle and to the legs.
5. Consider the circles $(x - 2)^2 + (y + 1)^2 = 16$ and $(x - 5)^2 + (y - 5)^2 = 1$.
 - a) Find the coordinates of the intersection point of the external common tangents of the circles.
 - b) Find the equations of the common tangents.
6. Which arithmetic sequences have the property that the quotient of the sum of the first $6n$ terms and the sum of the first $2n$ terms is the same for every positive integer n ?
7. Solve the equation

$$\log_{\frac{1}{3}}(9^x + a) + \log_3(2 \cdot 3^x) = 0$$

over the real numbers, where a is a real parameter.

8. Solve the following equation over the integers.

$$\sin\left(\frac{\pi}{3}\left(x - \sqrt{x^2 - 3x - 12}\right)\right) = 0$$