Problem Set II - 1995 (December 1995)

author: Imre Rábai
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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Consider the function $f(x) = (x-1)(x-2)(x-3)(x-4) + 10$, defined on the real numbers.
   a) Prove that $f(x) > 0$ for every real x.
   b) Where does $f(x)$ take its smallest value, and what is the smallest value?

2. One parallel side of the trapezoid $ABCD$ is $AB = 4.4$ units, the legs are $BC = DA = 1.7$ units, and the diagonals are $AC = BD = 3.9$ units. Find the area of the trapezoid.

3. The equations $x^2 + px - q = 0$ and $x^2 + qx - p = 0$ have one common root, and their other roots are different.
   a) Prove that the sum of the roots that are not common is 1.
   b) Using p, write down a quadratic equation whose roots are the two different roots in question.

4. Solve the following equation over the real numbers.
$$\frac{\sqrt{1+\sin 2x}}{\sin x + \cos x} = -1$$

5. Solve the following equation over the pairs of positive integers.
$$x^2 - 3xy + 2y^2 + 6 = 0$$

6. In triangle ABC (with the usual notation) $\gamma = 36^\circ$ and $a^2 - b^2 = 2cr$, where r is the radius of the circumscribed circle. Find the other two angles of the triangle.

7. Reflect the circle $x^2 + y^2 - 12x + 2y + 33 = 0$ in the line $x - 2y = -2$. Find the equations of the common tangents of the two circles.

8. Solve the following equation over the real numbers, where a is a real parameter.
$$\frac{x-a}{x-2} + \frac{10}{x+2} = \frac{44}{4-x^2}$$

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