

Problem Set V - 1996 (April 1996)

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. For which x are the following functions defined, and what are their ranges?

$$f_1(x) = x^2 - 9x - 22, \quad f_2(x) = |f_1(x)|, \quad f_3(x) = \sqrt{f_1(x)}, \quad f_4(x) = \log f_1(x)$$

2. The roots of the equation $x^2 - 10x + 21 = 0$ are the radii of the bases of two equilateral cylinders (cylinders whose height is equal to the diameter of the base). Without computing the roots, find the sum of the surface areas and the sum of the volumes of the cylinders.

3. Find the real numbers x for which

$$(\sqrt{x+1} - 2) \cdot (\sqrt{x+3} - 3) \cdot (\sqrt{x+5} - 4) \geq 0$$

4. The lines a, b, c with equations $2x + y + 9 = 0$, $-x + 2y + 8 = 0$, $3x + 4y + 6 = 0$ form a triangle. Find the equation of its circumscribed circle, and the radius of its inscribed circle.

5. In a plane, of the points S, Z, L , the points S and L are fixed and Z is moving, with $SL = 2$, $SZ = s$, $LZ = l$, and $2s^2 + 4 = l^2$. What is the set of the points Z ?

6. Solve the following equation.

$$36 \sin^4 x - 12 \sin^3 x - 17 \sin^2 x + 3 \sin x + 2 = 0$$

7. Is it true that the system of equations $xy = c$, $x - y = c$ has a solution over the rational numbers if and only if c is the reciprocal of the product of two consecutive integers (neither of them 0)?

8. The volume and the surface area of a right circular cone are in the ratio 7 : 4 (as numbers). Find the radius of the base, the height, and the slant height of the cone, if they are integers.