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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Let $a = \sin^2 1995^\circ$ and $b = \cos^2 1995^\circ$. Find the exact value of $\log_2 (ab)^{-\frac{1}{2}}$.
2. A cube and a regular tetrahedron have the same surface area. Find the ratio of their volumes.
3. The shortest altitude of an obtuse triangle is equal to the perpendicular projection of the shortest side onto the longest side. Express the radius of the circumscribed circle in terms of the sides of the triangle.
4. For the functions $f(x) = x^2 - 2x + 3$ and $g(x) = x^2 - 4x + 7$, defined on the real numbers, find all positive integers n for which $f(n)$ is the geometric mean of n and $g(n)$.
5. The real numbers a, b, c, d are not less than $\frac{1}{3}$, and $a + b + c + d = 2$. Prove that

$$\sqrt{6a-2} + \sqrt{6b-2} + \sqrt{6c-2} + \sqrt{6d-2} \leq 4$$

6. The vertices of a triangle are $A(5, -2)$, $B(-3, 2)$, $C(-2, -1)$. Find the equation of its circumscribed circle.
7. Six different real numbers are six consecutive terms of an arithmetic sequence. The first, the second, and the sixth terms are three consecutive terms of a geometric sequence. Find the ratio of the first term and the difference of the arithmetic sequence. Which term of the arithmetic sequence is the tenth term of the geometric sequence? Show that the arithmetic sequence contains every term of the geometric sequence.
8. Solve the following equation over the real numbers.

$$\tan^2 x + \tan^2 x \tan 2x \tan 3x - \tan 3x + \tan 2x - 6 \tan 3x \tan 2x - 6 = 0$$