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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. a) Solve the following equation over the natural numbers.

$$\log_2 3 + \log_{2^2} 3^2 + \cdots + \log_{2^n} 3^n = \log_2 81$$

- b) Find the real solutions of the following equation.

$$2\sqrt{3^{2x-2} + 9^x - 81} = 96 - 9^{x-1} - 3^{2x}$$

2. The legs a and b of a right triangle satisfy

$$2 \cdot \log \frac{a+b}{4} = \log a + \log b$$

Find the value of $\sin 2\alpha$, where α is the angle opposite a .

3. Prove that if the perpendicular bisector of a diagonal of a rectangle divides the longer side a of the rectangle into two parts, one of which is equal to the shorter side b , then $a + b$ is the hypotenuse of an isosceles right triangle with legs a .
4. In a triangle (with the usual notation) the angle γ is opposite the side c , and m_c is the altitude to the side c . Prove that

$$\frac{c}{\sin \gamma} > m_c$$

5. The vertices of the broken line $A_0 A_1 \dots A_n$ are given as follows:

$$A_{4k}(0, -2k), \quad A_{4k+1}(4k+1, 2k+1), \quad A_{4k+2}(-1, 2k+1), \quad A_{4k+3}(-4k-4, -2k-2)$$

where k is a nonnegative integer. How long is this broken line from A_0 to A_{1997} ?

6. The opposite edges of a tetrahedron are pairwise equal, and the three different edge lengths are $2\sqrt{85}$, $3\sqrt{41}$, $\sqrt{421}$ units. Find the distances between the pairs of opposite (skew) edges.
7. Three points A , B , C form a right triangle on the map, with the right angle at B . The difference in elevation between A and B is 80 meters, and between B and C it is 120 meters. A straight road with a 10° upward slope leads from A to B , and a straight road with a 12° upward slope leads from B to C . Find the straight-line distance between A and C . What is the angle ABC in reality?
8. Take the sum of the cubes of the first n positive integers and divide it by the sum of the squares of the first n positive integers. Can the quotient be an integer, if $n > 1$ is an integer?