

Problem Set III - 1997 (January 1997)

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In memory of István Bóka, teacher (István Bóka was the author's mathematics teacher in 1941–44 at the Baross Gábor High School in Szeged.)

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equations over the real numbers.

a) $9^x - 2^{x+\frac{1}{2}} = 2^{x+\frac{7}{2}} - 3^{2x-1}$ b) $9^x - 2^{x+\frac{1}{2}} = 2^{x+\frac{3}{2}} - 3^{2x-1}$

2. The terms a_1, a_2, a_4, a_5 of an arithmetic sequence satisfy $a_1 a_2 = 3$ and $a_4 a_5 = 63$. Find the first term and the difference of the sequence.
3. Prove that if $\sin(\alpha - \beta) = 0$, then $\sin(2\beta - \alpha) = \sin \alpha$.
4. Find the values of the parameter m for which one root of the equation

$$mx^2 + 2(m-1)x - 4 = 0$$

is less than 3, and the other root is greater than 3.

5. Find the legs of the right triangle whose hypotenuse is $4\sqrt{10}$ units, and whose angle bisector from the right angle is $3\sqrt{2}$ units long.
6. The circles k_1 and k_2 have equal radii, and their common points are $A(3, 2)$ and $B(7, 6)$. Find the equations of the circles, if the center of one of them is on the line $4x + 5y = 20$.
7. Solve the following equation over the pairs of real numbers.

$$x^2 + 4x \cos xy + 4 = 0$$

8. Find the largest and the smallest values of the function

$$x \mapsto f(x) = |x^2 - 6x| + |x^2 - 6x + 8|$$

if its domain is the set of real numbers satisfying $3 - \sqrt{10} \leq x \leq 7$. Where does the function take its extreme values?