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In memory of Imre Hajnal, teacher. Imre Hajnal (1926–1996) earned his diploma as a secondary school teacher of mathematics and physics at Eötvös Loránd University. He taught at the Bethlen Gábor High School in Hódmezővásárhely, at the Szakérttségis Kollégium in Szeged, and at the Ságvári Endre High School in Szeged, and also worked as a mathematics supervisor. He taught successfully in classes with a special mathematics curriculum. His high school textbooks, the teacher's guides written for them, and his other mathematics books will hopefully be used for a long time.

translated from: KoMaL, October 1997, pages 403-404

Pdf file: KoMaL, October 1997, pages 403-404

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The sum of two sides of a triangle is 14 units, and the angles on the third side are 30° and 70° . Find the lengths of the sides, the area of the triangle, and the radii of the circumscribed and the inscribed circles.
2. The first term of an arithmetic sequence is 30, and its difference is -3 . Find the term of the sequence that is one eighth of the sum of the terms before it.
3. One parallel side of the trapezoid $ABCD$ is $DC = 8$ units, the legs are $AD = 4\sqrt{3}$ and $BC = 2\sqrt{10}$ units, and $\angle BAD = 60^\circ$. Find the area of the trapezoid.
4. Solve the following system of equations over the pairs of real numbers.

$$2\sqrt{\frac{5x}{x-y}} + \sqrt{\frac{x-y}{5x}} = 3, \quad (y+4x)(x-y+20) = 0$$

5. Prove that the following relation holds in every triangle (with the usual notation).

$$\frac{b-c}{a} = \frac{\sin \frac{\beta-\gamma}{2}}{\sin \frac{\beta+\gamma}{2}}$$

6. A line e through the origin cuts off a chord of length 8 from the circle $(x-4)^2 + (y-8)^2 = 32$. Find the equation of the line e , and the coordinates of the endpoints of the chord.
7. For which values of the parameter a does the equation

$$\sin^4 x + \cos^4 x + \sin 2x + a = 0$$

have a solution? Solve the equation for $a = \frac{1}{2}$ and for $a = -\frac{3}{2}$.

8. The parallel lines e and f are $4\sqrt{2}$ units apart. We take the points A and B on the line e so that $AB = 3\sqrt{2}$, and a point C on the line f . Let P be a point of the segment AC ; the line BP meets the line f at D . How far from the line e is the point P , if the sum of the areas of the triangles ABP and PCD is as small as possible? What is this smallest area?