

Problem Set IV - 1998 (April 1998)

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equation over the real numbers.

$$\sqrt{x^2 - 10x + 30} + x^2 = 10x - 18$$

2. Seen from above, we see a regular hexagon $ABCDEF$ with side 10, but in reality the vertices are at heights 4, 6, 3, 6, 2, 3 units above the horizontal ground, in this order. Seen from above, the lines AD , BE , and CF seem to meet at one point. In reality, what are the vertical distances between these lines at the apparent intersection point?
3. The four-digit number $\overline{abbc}$ (in base ten) is a prime. Different letters denote different digits. The sum of the four digits is 26. If we cut the number into two parts with one cut, then among the numbers obtained, $\overline{abb}$, $\overline{c}$, $\overline{ab}$, $\overline{bc}$, and also $\overline{bbc}$ are primes. Find all four-digit numbers with these properties.
4. In the coordinate plane, the points $A(0, 0)$, $B(6, 0)$, $D(0, 6)$, $P(2, 0)$, $S(0, 4)$, $E(9, 0)$, $F(0, 9)$ are given. The point C is an interior point of the segment EF . The point of the segment CD dividing it into thirds that is closer to C is R , and the point of the segment BC dividing it into thirds that is closer to B is Q . The midpoints of the segments AC , BD , PR , QS are K , L , M , N . Find the ratio $KL : MN$ as a function of the coordinates of the point C .
5. The function f defined on the real numbers is given by $f(a) = 2a^3 - 3a^2 + a$. Solve the following equation over the real numbers.

$$f(\sin^2 x) + f(\cos^2 x) = 0$$

6. Solve the following equation over the real numbers.

$$\left(\frac{\sqrt{5}}{7}\right)^{\frac{2}{-5+\log x} - \frac{4}{1+\log x}} = 9.8$$

7. Two circles that are not in the same plane intersect each other in two points. Prove that there is always a sphere that contains both circles.
8. Three gold diggers found a nugget every day, which weighed as many grams as the number of the day they were working (on the first day 1 gram, on the second day 2 grams, and so on). On which day can they first share the gold equally, if they do not want to cut the nuggets?