Problem Set II - 1998 (December 1998)

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In honor of the Fazekas Mihály Practice High School of Budapest, where I taught between 1958 and 1966, and where we organized the first high school class with a special mathematics curriculum.

translated from: KoMaL, December 1998, page 514
Pdf file: KoMaL, December 1998, page 514

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The sides of a triangle are 8.8, 28.6, and 33 units long. Find the area of the triangle and the radius of its inscribed circle.

2. Solve the following equation over the real numbers.

$$\sqrt{11+x+6\sqrt{x+2}}+\sqrt{4-x+2\sqrt{3-x}}=7$$

3. Of six numbers, the first four are consecutive terms of a geometric sequence, and the last four are consecutive terms of an arithmetic sequence. Find the six numbers, if the sum of the last four is 16, and the product of the third and the sixth is -20.

4. Two sides of a triangle are 10 and 15 units long, and the bisector of the angle between them is $6\sqrt{3}$ units long. Find the angle between the two given sides and the length of the third side of the triangle.

5. Find the equation of the line that goes through the point $P\,(6,10)$, and whose segment between the lines $4x+3y=32$ and $4x+3y=2$ has a projection of length 2 onto the y-axis.

6. For which values of the real parameter m does the equation

$$\cos 2x+(7-5m)\sin x-3m^2+9m-7=0$$

have a solution? Solve the equation for $m=\dfrac{1}{3}$, 1, $\dfrac{5}{3}$, 2, and 3.

7. Solve the following inequality over the real numbers.

$$\log_4\frac{1}{x^2}+4\cdot\log_x\frac{1}{16}+10\geq 0$$

8. For which values of the real parameter a does the equation

$$x^4-(3a+2)\,x^2+a^2=0$$

have four different roots that are four consecutive terms of an arithmetic sequence?

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