Problem Set III - 1998 (February 1998)

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translated from: KoMaL, February 1998, page 89
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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equation over the real numbers.

$$5^x \cdot 2^{\frac{x}{x-1}} = 100$$

2. Prove that if in a triangle

$$\frac{a-b}{a} = 1 - 2\cos\gamma$$

then the triangle is isosceles.

3. Solve the following inequality over the real numbers.

$$\frac{x^2}{\left(2-\sqrt{x+4}\right)^2} > x+6$$

4. For which values of the variables do the following expressions take their largest values? Find these largest values.

a) $\dfrac{4\sqrt{2}+2}{2^{x-1}+2^{2-x}+1}$ b) $\dfrac{32}{(3-x+\log 2y)^2+x^4-2x^2+17}$

5. The difference of an arithmetic sequence is $\frac{1}{2}$. The sum of the first n terms is $\frac{75}{2}$, and the sum of the first $n+3$ terms is 63. Find n and the first term of the sequence.

6. Solve the following system of equations over the triples of integers.

$$x^2-y^2-z^2=1, \qquad y+z-x=3$$

7. The circle k is tangent to the x-axis, and also to the circle $(x-17)^2+(y+17)^2=100$ at its point with first coordinate $x_0=11$. Find the equation of the circle k.

8. Depending on the value of the real parameter m, how many different real roots does the following equation have?

$$x^4+2\left(m^2-5\right)x^2+m^4-10m^2+7=0$$

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