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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equation over the real numbers.

$$\frac{x-2}{x-1} + \frac{x+2}{x+1} = \frac{x-4}{x-3} + \frac{x+4}{x+3} - \frac{28}{15}$$

2. Prove that in every triangle

$$\sin^2 \alpha = \sin^2 \beta + \sin^2 \gamma - 2 \sin \beta \sin \gamma \cos \alpha$$

where α, β, γ are the angles of the triangle.

3. Find the values of the parameter a for which the inequality

$$-2 < \frac{x^2 + ax - 2}{-x^2 + x - 1} < 3$$

holds for every real number x .

4. The base of the isosceles triangle ABC is $BC = 4.8$ units, and the altitude to it is $m = AD = 4.5$ units. The point P is the point of the leg AC for which the quadrilateral $ABDP$ is a cyclic quadrilateral. Find the length of the segment AP .

5. The sums of the first n , k , and $n+k$ terms of an arithmetic sequence are S_n , S_k , and S_{n+k} . Prove that

$$(n+k)(S_n - S_k) = (n-k)S_{n+k}$$

6. Two vertices of triangle ABC are $A(0, 5)$ and $B(-4, 0)$, and the area of the triangle is 9 square units. Find the first coordinate of the vertex C , if its second coordinate is 7.

7. Find the values of the parameter a for which one root of the equation

$$2x^2 - 2(2a+1)x + a(a-1) = 0$$

is less than a , and the other root is greater than a .

8. For which values of the parameter α does the equation

$$x^2 - 2(\sin \alpha + \cos \alpha)x + 1 + 2\cos^2 \alpha = 0$$

have a) two equal, b) two different real roots?