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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. For which value of n are a, b, c_n three consecutive terms of an arithmetic sequence, if

$$a = \tan \frac{0 \cdot \pi}{5} + \tan \frac{1 \cdot \pi}{5} + \tan \frac{2 \cdot \pi}{5} + \tan \frac{3 \cdot \pi}{5} + \tan \frac{4 \cdot \pi}{5} + \tan \frac{5 \cdot \pi}{5}, \quad b = \sin^{\cos 60^\circ} 60^\circ + \cos^{\sin 30^\circ} 30^\circ$$

and $c_1 = 3, c_n = 2\sqrt{c_{n-1}}$ for $n \geq 2$?

2. In the quadratic equation $5x^2 - 6(a+b)x + a^2 + b^2 + 2ab = 0$ the real parameters a and b satisfy $1 < a + b < 5$. Show that one root of the equation is greater than 0 but less than 1, and the other root is greater than 1. Is the converse of this statement true?

3. Find the integers n and m for which

$$A = n^5 - 5n^3 + 4n - 2^m + 1$$

is divisible by 5.

4. The center of the inscribed circle of a right triangle is at distances x, y, z from the vertices (z is the distance from the vertex of the right angle). Show that one angle of the triangle with sides xy, xz , and yz is 135° .
5. Find the equation of the circle that the circles $(x+2)^2 + (y-5)^2 = 1$, $(x-1)^2 + (y-6)^2 = 1$, and $(x-4)^2 + (y+3)^2 = 1$ touch from the inside.
6. The numbers a, b , and c are (not necessarily consecutive) terms of an arithmetic sequence of positive numbers, and

$$\frac{c-b}{a} + \frac{a-c}{b} + \frac{b-a}{c} = 0$$

Find the sum of the first n terms of the sequence, if we know its first term.

7. Consider the functions $f(x) = \sin^4 x + \cos^4 x$ and $g(x) = \cos x$. Give a geometric transformation that takes the graph of g to the graph of f .
8. Solve the following system of equations over the real numbers.

$$\sqrt[3]{x-72} + \sqrt[3]{y+1998} = 25, \quad x+y = 1999$$