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In memory of Gyula Komlós, teacher (my friend)

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance. The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The medians of a triangle are 2.4, 2.6, and 1 units long. Find the area of the triangle.

2. Find the values of the real parameter a for which one root of the equation

$$x^2 - 4x + (3 + 2a - a^2) = 0$$

is twice the other root.

3. Solve the following system of equations over the pairs of real numbers.

$$x^{\log_2 y} + y^{\log_2 x} = 2, \quad \log_4 x - \log_4 y = \frac{1}{2}$$

4. For which values of the real parameter m is the value of the expression

$$(5 - m)x^2 - 3x + m + 3$$

positive for every real number x ?

5. Consider the triangles whose area is $T = 4\sqrt{3}$ square units and one of whose angles is 60° .

a) In which of these triangles is the sum of the sides enclosing the 60° angle the smallest? What are the sides and angles of this triangle?

b) In which of these triangles is the side opposite the 60° angle the shortest?

6. Let $A(1, 0)$, $B(0, 4)$, and $C(c, 6)$. Find the value of c , if the area of the triangle is 13 square units.

7. Solve the following inequality over the real numbers.

$$\sin^4 x + \cos^4 x \geq \frac{5}{8}$$

8. Solve the equation $\sqrt{a-x} + \sqrt{-x} = 1$ over the real numbers, where a is a real parameter.