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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. One parallel side of a trapezoid is 22 units long, its legs are 14.3 and 16.5 units long, and its height is 13.2 units. Find the area of the trapezoid.
2. Prove that if n is a positive integer, then the following expressions are divisible by 24.
 - a) $(n^2 + 3n + 1)^2 - 1$
 - b) $n^4 + 6n^3 + 11n^2 + 6n$
3. Find the places of the extreme values and the range of the function $x \mapsto \frac{1}{\cos^2 x + \sin^4 x}$ (x real).
4. The numbers a_1, a_2, a_3, a_4 and b_1, b_2, b_3, b_4 are the first four terms of two geometric sequences, with $a_1 = b_1$, $a_2 - b_2 = 3$, $a_3 - b_3 = 6$, and $a_4 - b_4 = 36$. Write down the first four terms of the sequences.
5. Solve the following inequality over the real numbers.

$$\sqrt{\log_2 16 - \log_2 x^5 + (\log_2 x)^2} \leq \sqrt{2(\log_2 x)^2 - 5 \log_2 x}$$

6. Solve the following equation over the real numbers.

$$\left| \frac{4 \sin^2 x - 1}{\cos x} \right| = \frac{4 \cos^2 x - 3}{\cos x}$$

7. The parallel sides of the trapezoid $ABCD$ are AB and CD , $AB = 3 \cdot CD$, $BC = AD$, and the diagonal AC is perpendicular to the diagonal BD . Find the coordinates of the vertices A and B , if $C(6, 4)$ and $D(3, 3)$.
8. Consider the inequality

$$((a + 2)x^2 + (1 - a)x + a - 1)(4x - x^2 - 5) \leq 0$$

where a is a real parameter.

- a) Find the value of a for which the inequality has exactly one solution.
- b) For which a does the inequality have infinitely many solutions, and for which a does it have no solution at all?