

Technical Colleges Entrance Exam - 1986

translated from: KoMaL, September 1986, pages 251-252

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These problems were given at the written mathematics entrance examination of the technical colleges in Hungary in 1986. Problems 6/a, 7/a, and 8/a were recommended for students coming from vocational secondary schools (in place of problems 6, 7, and 8).

1. Solve the given equation over the real numbers.

$$\log(4x + 2) - \log(-6x) = \log(1 - 2x)$$

2. The first term of an arithmetic sequence is -210 , and its n th term is 228 . The sum of the terms between them is 45 . Write down the first n terms of the sequence.
3. In triangle ABC , the angle at vertex A is a right angle, and one of the angles is 30° . The distance of vertex A from the center of the inscribed circle is $2\sqrt{2}$. Find the radius of the circumscribed circle.
4. For which real numbers x are the following expressions defined? What values do they take?

a) $\sin\left(\frac{\pi}{6} \cos(\pi x)\right)$ b) $\sqrt{\sin(\pi x)}$

5. Two sides of a triangle are 8 and 12 , and the median drawn to the third side is 9 . Find the area of the triangle.
6. Solve the given inequality over the real numbers.

$$(x^2 - x + 1)^{x-2} > 1$$

7. The base of a regular square pyramid is the square $ABCD$, its apex is O , and all of its edges are h units long. Find the area of the quadrilateral that the plane through the base edge AB and the midpoint F of the edge OC cuts out of the pyramid. What angle does this plane make with the base?
8. We write the consecutive positive integers on a board, starting with 1 , up to a certain number. Then we erase one of the numbers. The arithmetic mean of the remaining numbers is $\frac{602}{17}$. Which number did we erase?
- 6/a. The height of a trapezoid inscribed in a circle is 8 , and one of its legs is seen from the center of the circle at an angle of 60° . Find the area of the trapezoid.
- 7/a. The line g through the origin cuts a chord of length 10 from the circle $(x - 5)^2 + (y - 10)^2 = 50$. Find the equation of g and the coordinates of the endpoints of the chord.
- 8/a. Solve the given equation over the real numbers.

$$\sqrt{x + 2 + 4\sqrt{x - 2}} + \sqrt{x + 7 - 6\sqrt{x - 2}} = 5$$

Economics Entrance Exam - 1986

translated from: KoMaL, October 1986, pages 298-299

Pdf file: KoMaL, October 1986, pages 298-299

These problems were given at the written mathematics entrance examination of the universities and colleges of economics in Hungary in 1986. Problems 6/a, 7/a, and 8/a were recommended for students coming from vocational secondary schools (in place of problems 6, 7, and 8).

- Find n if the ratio of the sum of the first n positive even numbers and the sum of the first n positive odd numbers is $\frac{21}{20}$.
- Solve the given system of equations over the real numbers.

$$\begin{aligned} 8^{2x+1} &= 32 \cdot 2^{4y-1} \\ 5 \cdot 5^{x-y} &= \sqrt{25^{2y+1}} \end{aligned}$$

- For which real numbers are the following expressions defined?

a) $\log \log \sqrt{x}$ b) $\log \sqrt{\log x}$ c) $\sqrt{\log \log x}$

- In rectangle $ABCD$, $AB = 2.4 \cdot BC$. We enlarge the rectangle by a dilation with center A so that the area of the new rectangle is $\frac{9}{4}$ times the area of the original one. Because of the enlargement, the diagonal of the rectangle becomes 13 units longer. Find the sides of rectangle $ABCD$.
- The base ABC of the triangular pyramid $OABC$ is an isosceles right triangle with $\angle ACB = 90^\circ$. The lateral edge $OC = 3\sqrt{3}$ is perpendicular to the plane of the base. The face OAB makes an angle of 60° with the plane of the base. Find the surface area and the volume of the pyramid.
- Let $ABCD$ be a quadrilateral in the plane. Give a construction for the point P for which $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} + \overrightarrow{PD} = \mathbf{0}$.
- How should we choose p and q in the polynomial $x^2 + px + q$ if we want to get the polynomial $x^4 + 1$ when we multiply it by a suitably chosen quadratic polynomial?
- Solve the given equation over the set of integers.

$$\cos \left(\frac{\pi}{4} \left(3x - \sqrt{9x^2 - 16x - 80} \right) \right) = 1$$

- 6/a. A parabola with an axis parallel to the y -axis, opening upward, passes through the point $A(5, 4)$ and is tangent to the x -axis. The tangent line of the parabola at A is perpendicular to the vector $\mathbf{v}(4, -1)$. Find the equation of the parabola.

- 7/a. Prove that a triangle is a right triangle if and only if its acute angles α and β satisfy $\sin \alpha + \sin \beta = \cos \alpha + \cos \beta$.

- 8/a. Prove that every term of the sequence

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n}$$

is greater than the previous term, but less than 1.

Entrance Exam for Universities Abroad - 1986

translated from: KoMaL, November 1986, page 360

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These problems were given in 1986 at the written mathematics entrance examination of Hungarian students applying to study at universities abroad. Problems 1–5 were the same for everyone. Of problems 6–8, the ones marked A were recommended for students who graduated from academic high schools, and the ones marked B for students who graduated from vocational secondary schools.

1. Solve the given equation over the real numbers.

$$\log(36^{5x} + 792) = \log 16 + \log 63$$

2. In a right triangle, the bisector of one of the acute angles divides the opposite side into two segments of length 1 and 2. Find the angles and the sides of the triangle.

3. Solve the given system of equations over the real numbers.

$$\sqrt{x + 3y + 1} = 2, \quad \sqrt{2x - y + 2} = 7y - 6$$

4. Three vertices of parallelogram $ABCD$ are $A(-3, -2)$, $B(2, 0)$, and $C(3, 3)$. The line through B perpendicular to the diagonal AC intersects the line CD at E . Find the coordinates of E .

5. Of four numbers, the first three are consecutive terms of a geometric sequence, and the last three are consecutive terms of an arithmetic sequence. The sum of the first and the fourth number is 14, and the sum of the second and the third is 12. Find the four numbers.

- 6A. Solve the given system of inequalities over the real numbers.

$$x^2 - 6x - 7 < 0, \quad \sin x > 0$$

- 7A. Let $ABCD$ and $EFGH$ be two parallelograms in the same plane. Prove that the midpoints of the segments AE , BF , CG , and DH , if they are not on one line, are the vertices of a parallelogram.

- 8A. Prove the following statement for every positive integer n : if a , b , c are integers such that n divides $a + b + c$ and $a^2 + b^2 + c^2$, then n also divides $a^4 + b^4 + c^4$.

- 6B. In the isosceles triangle ABC , the base BC is 10 units long, the altitude from A is 12, and D is the midpoint of the base. Let M be a point of the leg AC . Find the length of the segment AM if the quadrilateral $AMDB$ is cyclic.

- 7B. Solve the given system of equations over the real numbers.

$$\frac{xy}{x+y} = 1 - z, \quad \frac{yz}{y+z} = 2 - x, \quad \frac{zx}{z+x} = 2 - y$$

- 8B. Prove that if $0 \leq x < \frac{\pi}{2}$, then

$$\frac{\sin x}{1 + \tan x} \leq \frac{\sqrt{2}}{4}$$