

Written Final and Entrance Exam - 1988

translated from: KoMaL, September 1988, pages 251-252

Pdf file: KoMaL, September 1988, pages 251-252

These problems were given at the joint written final (érettségi) and entrance examination in mathematics in Hungary in 1988. Problems 1–5 were the same for all students; for problems 6–8, there was one version (A) recommended for students from academic high schools and another one (B) for students from vocational secondary schools. The points for each problem are shown in parentheses.

For all students

1. Solve the given system of equations over the real numbers.

$$\frac{4}{x} + \frac{5}{y} = 23, \quad \frac{5}{x} - \frac{4}{y} = -2$$

(9 points)

2. Solve the given equation over the real numbers.

$$\left(\frac{3}{4}\right)^{\log x} + \left(\frac{4}{3}\right)^{\log x} = \frac{25}{12}$$

(11 points)

3. A rhombus with a 30° angle is drawn around a circle of area t . Express the area of the rhombus in terms of t . (12 points)

4. For which real numbers x are the following expressions defined?

$$\text{a) } \sqrt{3 - \sqrt{x-2}} \quad \text{b) } \sqrt{x - \sqrt{x-2}} \quad \text{c) } \sqrt{1 - \tan^2 x}$$

(13 points)

5. In triangle ABC , the angle at vertex A is twice the angle at vertex B , twice the side BC is equal to three times the side AC , and the area of the triangle is 27 times the sine of the angle at vertex C . Find the angles and the sides of the triangle. (13 points)

A. Recommended for students from academic high schools

6. Find the equation of the circle whose center is on the x -axis, and which is tangent from the inside to the circle k_1 : $x^2 + y^2 + 4x + 6y - 87 = 0$, and tangent from the outside to the circle k_2 : $x^2 + y^2 - 20x - 12y + 111 = 0$. (13 points)

7. The base of a pyramid is a square, and its lateral faces are congruent isosceles triangles. The height of the pyramid makes a 30° angle with the lateral faces. A cube is placed inside the pyramid so that four vertices of the cube are on the base of the pyramid, and the other four vertices are on the lateral faces, on the altitudes of the lateral faces drawn from the apex of the pyramid. Find the ratio of the volumes of the pyramid and the cube. (14 points)

8. Let p and q be positive integers with $q \geq p$. Prove that

$$\frac{q + \frac{1}{2}}{p - \frac{1}{2}} > \frac{1}{p} + \frac{1}{p+1} + \frac{1}{p+2} + \cdots + \frac{1}{q}$$

(15 points)

Problem Sets from 1988

B. Recommended for students from vocational secondary schools

6. From the points P_1 and P_2 of the parabola $x - 2 = (y - 1)^2$, the segment with endpoints $A(2, 1)$ and $B(6, 1)$ is seen at a right angle. Find the area of the quadrilateral AP_1BP_2 . (13 points)
7. A right circular cylinder has height 2 and radius 7. A square is inscribed in the cylinder so that its plane intersects the axis of the cylinder, and its vertices are on the circles of the base and of the top. Find the side of the square and the angle between the axis of the cylinder and the plane of the square. (14 points)
8. Solve the given inequality over the real numbers.

$$2\sqrt{|1 - 2x|} \geq 1 + x$$

(15 points)

Technical Universities Entrance Exam - 1988

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These problems were given at the written mathematics entrance examination of the technical universities and colleges in Hungary in 1988. Problems 1–5 were the same for all applicants; for problems 6–8, there was one version recommended for students from academic high schools and another one for students from vocational secondary schools. The points for each problem are shown in parentheses.

For all applicants

- Find the exact values of the given expressions without using approximate values.

$$A = 10^{1-\log 2} + 3^{\log_9 36}, \quad B = 125^{-\frac{1}{3}} \cdot \sqrt[3]{1000} \cdot \log_{\sin \frac{\pi}{6}} \sqrt{2}, \quad C = \sqrt{\log \tan 36^\circ + \log \tan 54^\circ}$$

(9 points)

- Let AB be a diameter of a circle of radius 1, and let C be the point on the tangent line to the circle at A for which the segment AC has length $2\sqrt{3}$. Find the area of the common part of the triangle ABC and the disk. (11 points)
- We build a large cube out of small cubes with unit edges. If we build the largest possible cube from the small cubes we have, then 107 small cubes are left over. If we want one more small cube along each edge, then we would need 62 more small cubes. How many small cubes do we have? (12 points)
- For which pairs of real numbers is the following system of equations true?

$$\begin{aligned} y - x &= 44 \\ \sqrt{\frac{6x}{x+y}} + \sqrt{\frac{x+y}{6x}} &= \frac{5}{2} \end{aligned}$$

(13 points)

- The line $x + y = 30$ intersects the x -axis at A and the y -axis at B , and let O be the origin. Find the equation of the line e that is parallel to the line $x - 3y = 6$, intersects the segment OB at C and the segment AB at D , and for which the area of the quadrilateral $OADC$ is 234 units. (13 points)

Recommended for students from academic high schools

- For given real numbers p and q , find the first term and the difference of every arithmetic sequence in which the sum of the first four terms is p , and the quotient of the fourth term and the first term is q . (13 points)
- Solve the given equation, if p is a prime number and n is a positive integer.

$$1 + p + p^2 + \cdots + p^n = 2801$$

(14 points)

- Consider four vertices of a unit cube such that no two of them are on the same edge of the cube. The tetrahedron determined by these four points is cut by planes parallel to two opposite edges of the tetrahedron. Considering all such planes, what are the largest possible perimeter and the largest possible area of the cross-section? (15 points)

Recommended for students from vocational secondary schools

Problem Sets from 1988

6. The difference of the volumes of two right circular cylinders is 48π , and the difference of their lateral surface areas is 24π . The radius of the cylinder with the larger volume is 1 more than the radius of the other one, and its height is $\frac{4}{3}$ times the height of the other one. Find the heights and the radii of the cylinders. (13 points)

7. Solve the given equation over the pairs of real numbers.

$$2 \cos^2 \frac{x^2 + 3y}{6} = 3^x + 3^{-x}$$

(14 points)

8. In a geometric sequence of positive numbers, the sum of the first n terms is S , and the sum of the reciprocals of the first n terms is R . Express the product of the first n terms in terms of S , R , and n . (15 points)

Economics Entrance Exam - 1988

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Pdf file: KoMaL, October 1988, pages 301-302

These problems were given at the written mathematics entrance examination of the universities and colleges of economics in Hungary in 1988. Problems 1–5 were the same for all applicants; for problems 6–8, there was one version recommended for students from academic high schools and another one for students from vocational secondary schools. The points for each problem are shown in parentheses.

For all applicants

1. Solve the given equation over the real numbers.

$$\sqrt{1 + x\sqrt{x^2 - 24}} = x - 1$$

(9 points)

2. The sum of the first three terms of an arithmetic sequence is 105. If we leave the first two numbers unchanged and add 180 to the third number, then we get the first three terms of a geometric sequence. Find the first three terms of the arithmetic sequence. (11 points)

3. Which real numbers satisfy the given equation?

$$\log(4^{x-2} + 9) - \log(2^{x-2} + 1) + \log 2 - 1 = 0$$

(12 points)

4. One angle of a triangle is equal to the sum of the other two angles. The radius of the circle circumscribed around the triangle is 6 units, and the area of the square drawn on the shortest side is equal to the area of the triangle. Find the area of the triangle. (12 points)

5. In triangle ABC , $AB = BC$. Let F be the midpoint of BC , and let T be the foot of the altitude to the side BC . Find the coordinates of the vertices B and C , if $A(-2, 1)$, $T(16, 1)$, and the second coordinate of F is 13. (14 points)

Recommended for students from academic high schools

6. Let x, y, z be real numbers such that $xyz = 1$ and $1 + z + zx \neq 0$. Prove that the expression

$$\frac{1}{1 + x + xy} + \frac{1}{1 + y + yz} + \frac{1}{1 + z + zx}$$

is defined, and its value does not depend on the choice of x, y, z .

(12 points)

7. The tangents of the angles of a triangle are in the ratio 1 : 2 : 3. What is the ratio of the sides of the triangle? (15 points)

8. Let P be an arbitrary point in the plane of the quadrilateral $ABCD$. The reflection of P through A is Q , and the reflection of P through B is R . Prove that the quadrilateral $ABCD$ is a parallelogram if and only if the reflection of Q through D is the same point as the reflection of R through C . (15 points)

Recommended for students from vocational secondary schools

6. In rectangle $ABCD$, $AB = 2a$ and $BC = a$. Circles are drawn on the sides AB and AD as diameters; their intersection point inside the rectangle is M . Show that M is on the diagonal BD . How long are the segments MA , MB , and MD ? (13 points)

7. Solve the given equation over the real numbers.

$$x^2 + 2x \sin xy + 1 = 0$$

(14 points)

8. Prove that if the sum of the real numbers a and b is 1, then

$$a^4 + b^4 \geq \frac{1}{8}$$

(15 points)