

Written Final and Entrance Exam - 1990 (I)

translated from: KoMaL, December 1990, pages 442-443

Pdf file: KoMaL, December 1990, pages 442-443

These problems were given at the joint written final (érettségi) and entrance examination in mathematics in Hungary in 1990. KoMaL published them in December 1990 as practice set IV ("Matematika felvételre előkészítő feladatok - 1990. - IV.") of its series for students preparing for the entrance examination.

1. Solve the given equation over the real numbers.

$$x^2 + 3x + |x + 3| = 0$$

2. The price of a product was first raised by 10%, then lowered by 10%. Finally it was raised by 10% again. What percent of the original price is the final price?
3. Find the area of triangle ABC in which $AB = 3$, $BC = 7$, and the median from vertex B is 4 units long.
4. Solve the following system of equations over the pairs of real numbers.

$$\begin{aligned}x^{-1} + y^{-1} &= x + y \\ (2 + \log y) \log x &= 1\end{aligned}$$

5. The parallel sides of trapezoid $ABCD$ are AD and BC . A circle is drawn on the side BC of the trapezoid as diameter. This circle is tangent to the side AD and bisects both diagonals. Find the angles of the trapezoid.
6. Find the equation of the line that goes through the point $(6, 4)$ and intersects the lines $x + y = 4$ and $x + y = 5$ at points whose first coordinates differ by 2.
7. Solve the following equation over the real numbers.

$$\log_5 (x - 4) + \log_{\sqrt{5}} (x^3 - 2) + \log_{0.2} (x - 4) = 4$$

8. Find all pairs of real numbers (a, b) for which the equation

$$\cos(ax + b^2) - (a \cdot \cos x + b^2) = 1 - a$$

holds for every real number x .

Written Final and Entrance Exam - 1990 (II)

translated from: KoMaL, January 1991, pages 7-8

Pdf file: KoMaL, January 1991, pages 7-8

These problems were also given at the joint written final (érettségi) and entrance examination in mathematics in Hungary in 1990 (this is a second problem set from that year). KoMaL published them in January 1991 as practice set V ("Matematikai felvételtire előkészítő feladatok - 1991. - V."). The points for each problem are shown in parentheses.

1. The legs of a right triangle are 3 and $3\sqrt{3}$ units long. Find the perimeter and the area of the circular segment that the shorter leg of the triangle cuts off from the circle circumscribed around the triangle. (The circular segment is the one that is smaller than a half disk.) (10 points)
2. Solve the given equation over the real numbers.

$$\cot x - \sin 2x = \cot x \cdot \cos 2x$$

(10 points)

3. Four siblings are asked about their ages. The oldest one says: "Our birth years are the first four terms of an arithmetic sequence." The next one in age says: "15 years ago, the ages of my siblings were equal to the first three terms of a geometric sequence, and now the sum of their ages is $\frac{11}{4}$ times my age." Find the present ages of the siblings. (12 points)
4. The side AB of a triangle is 10, the median to it is 6, and another median is 9 units long. Find the sides AC and BC of the triangle. (12 points)
5. The points $A(-4, -1)$ and $B(6, 1)$ are on the circle $x^2 + y^2 - 2x - 25 = 0$. The chords AC and BC of the circle are in the ratio 3 : 2. Find the coordinates of the point C . (13 points)
6. A frustum of a cone is circumscribed around a sphere so that the base circle and the top circle of the frustum each touch the sphere at one point, and its lateral surface touches the sphere along a circle. What is the quotient of the volume and the surface area of the frustum, and what is the product of the radii of the base and top circles, if the radius of the sphere is 6 units? (14 points)
7. For what values of the real parameter p does the equation

$$1 + \frac{x}{x-p} = \frac{2p}{x+p} + \frac{2x^2 - 4x + 9p^2}{x^2 - p^2}$$

have a solution? For which values of p is the root of the equation less than p ?

(14 points)

8. Prove that if n is a positive integer, then
 - a) 81 is a divisor of $10^n(9n-1)+1$,
 - b) 1990 is a divisor of $(600^n - 3^n)(n^5 - n)$.

(15 points)

Written Final and Entrance Exam - 1990 (III)

translated from: KoMaL, February 1991, pages 63-64

Pdf file: KoMaL, February 1991, pages 63-64

These problems were also given at the joint written final (érettségi) and entrance examination in mathematics in Hungary in 1990 (this is a third problem set from that year). KoMaL published them in February 1991 as practice set VI ("Matematikai felvételtire előkészítő feladatok - 1991. - VI.").

1. Solve the following system of equations over the real numbers.

$$\frac{3}{\sqrt{x-2}} - \frac{5}{\sqrt{y+1}} = -\frac{1}{6}, \quad \frac{4}{\sqrt{x-2}} + \frac{2}{\sqrt{y+1}} = \frac{8}{3}$$

2. Solve the following equation over the real numbers.

$$729 \cdot 3^x - 2^{2x+12} = 0$$

3. For which real numbers x are the following expressions defined?

$$\text{a) } \frac{\sqrt{-x^2 + 8x - 16}}{x + 4} \quad \text{b) } \log \left| \frac{1 - 3^x}{3^x} \right| \quad \text{c) } \frac{1}{\sqrt{\log(1 - \cos^2 x)}}$$

4. We deposit one hundred thousand forints in a bank, at 16% annual interest. After four years, we withdraw 120 thousand forints. After that, how long do we have to wait until our deposit is more than one hundred thousand forints again?

5. Solve the equation

$$(1 + \sin 2x) \cdot (\cos x - \sin x) = \cos x + \sin x$$

over the real numbers.

6. Two adjacent vertices of rectangle $ABCD$ are $A\left(\frac{3}{2}, 1\right)$ and $B\left(\frac{9}{2}, 0\right)$. The ratio of the lengths of the sides AB and BC is $1 : 3$. Find the coordinates of the points C and D .

7. For what values of the parameter k does the quadratic equation

$$x^2 - 2(3k + 5)x + k^2 + 13k - 90 = 0$$

have two real roots with opposite signs?

8. Solve the following inequality over the real numbers.

$$\log_{\frac{2}{3}}(\log_{16}(x^2 - 48)) \geq 0$$