

Economics Entrance Exam - 1991

translated from: KoMaL, September 1991, page 253

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These problems were given at the written mathematics entrance examination of the universities and colleges of economics in Hungary in 1991. The points for each problem are shown in parentheses.

1. A factory makes a part in two quality classes. 78% of the revenue comes from the first class product. By what percent does the revenue of the factory increase if the production of the first class product is increased by 27% and the production of the second class product by 22%? (8 points)

2. Find the exact values of the following expressions (without using tables or a calculator).

a) $\log_3 \left(-\tan \frac{1991\pi}{4} \right)$ (3 points)

b) $1991\sqrt{\cos^2 9\pi - 1}$ (3 points)

c) $\log_{2+\sqrt{3}} (2 - \sqrt{3})$ (5 points)

3. The radii CA and CB of a circle with center C and radius 3 units make a 120° angle with each other. A cone is placed on the plane of the circle so that its base circle is tangent to the arc AB and to the segments CA and CB . The height of the cone is equal to the length of AB . Find the volume of the cone. (12 points)

4. Solve the given equation.

$$8 \sin^2 \frac{x}{2} - 2 \cos x = 7$$

(13 points)

5. Let $a_1, a_2, a_3, \dots, a_n, \dots$ be an arithmetic sequence. Prove that the sequence $b_1, b_2, b_3, \dots, b_n, \dots$ defined by $b_n = a_{n+1}^2 - a_n^2$ is also an arithmetic sequence. (13 points)

6. The axis of symmetry of the kite $ABCD$ is the diagonal AC , where $A(0, 0)$ and $C(8, 10)$. The area of the kite is 41 square units. One diagonal divides the other in the ratio 3 : 2, counting from the origin. Find the coordinates of the missing vertices. (13 points)

7. Which triples of real numbers satisfy the following system of equations?

$$\begin{aligned} x + y + z &= 2 \\ 2xy &= z^2 + 4 \end{aligned}$$

(14 points)

8. Solve the following equation over the real numbers.

$$\log_{1991}(x-3) + \log_{1992}(x-3) = 3 - \log(x^5 - 24)$$

(16 points)

Technical Universities Entrance Exam - 1991

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These problems were given at the written mathematics entrance examination of the technical universities and colleges in Hungary in 1991. The points for each problem are shown in parentheses.

1. Solve the following equation over the real numbers.

$$\frac{\sqrt{x+1}+2}{\sqrt{x+1}-1} = \frac{x+1}{x-2}$$

(10 points)

2. The third term of an arithmetic sequence is the square of the first term, and the sum of the first three terms is 30. Write down the first three terms of the sequence. (10 points)
3. The base BC of the isosceles triangle ABC is 40 units long, and the height to the base is 15 units. The feet of the perpendiculars from the midpoint F of the base to the legs are P and Q . Find the perimeter and the area of the quadrilateral $APFQ$. (12 points)
4. One vertex of a rhombus is $A(5, 8)$, and the equation of the line of the diagonal BD is $x - 2y + 6 = 0$. The side of the rhombus is 5 units. Find the coordinates of the other vertices and the area of the rhombus. (13 points)
5. Solve the following system of equations over the pairs of real numbers.

$$\frac{x}{y} + \frac{y}{x} = \frac{5}{2}; \quad \log_3(x-y) + \log_3(x+y) = 1$$

(13 points)

6. All edges of a regular square pyramid are equal. A cube is inscribed in the pyramid so that its base is on the base of the pyramid, and the vertices of its top face are on the lateral edges of the pyramid. How many times the volume of the cube is the volume of the pyramid? (13 points)
7. Prove that for any value of the real parameter p , the following equation has exactly one real root.

$$x|x+2p| = p$$

(14 points)

8. Let a , b , and c be the lengths of the sides of a triangle. Prove that

$$\frac{1}{3} \leq \frac{a^2 + b^2 + c^2}{(a+b+c)^2} < \frac{1}{2}$$

(15 points)