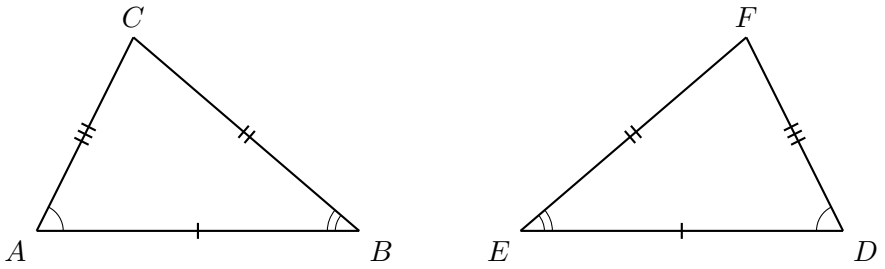
Two triangles are **congruent** if they have exactly the same shape and the same size. In other words, if we cut one of them out of paper, we can place it exactly on top of the other one. We are allowed to slide it, turn it, and even flip it over.



The two triangles in the picture are congruent: the second one is the mirror image of the first one. Equal sides are marked with the same number of tick marks, and equal angles with the same number of arcs. When we place one triangle on the other, A lands on D, B lands on E, and C lands on F. These are **corresponding vertices**.

Definition. Triangles ABC and DEF are **congruent** if their corresponding sides are equal and their corresponding angles are equal:

$$AB = DE, \quad BC = EF, \quad CA = FD, \quad \angle A = \angle D, \quad \angle B = \angle E, \quad \angle C = \angle F.$$

We write $\triangle ABC \cong \triangle DEF$.

Be careful. The order of the letters matters: it tells us which vertices correspond to each other. In our picture, $\triangle ABC \cong \triangle DEF$ is correct, but $\triangle ABC \cong \triangle EDF$ is not, because $AB = DE$, but $\angle A$ is not equal to $\angle E$.

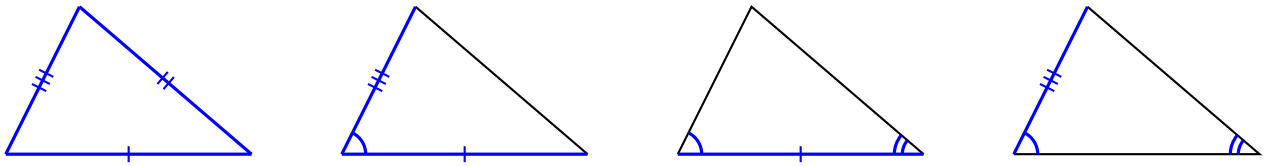
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How Can We Tell That Two Triangles Are Congruent?

To check the definition, we would need to compare six things: three sides and three angles. Fortunately, we do not have to. A triangle is already completely determined by just three of these parts, if we choose them well. The four most important cases are listed below. In each picture, the parts we compare are shown in blue.

Congruence Criteria. Two triangles are congruent if any of the following is true.

1. **SSS** (side-side-side): all three sides of one triangle are equal to the corresponding sides of the other.
2. **SAS** (side-angle-side): two sides and the angle *between* them are equal to the corresponding parts of the other.
3. **ASA** (angle-side-angle): two angles and the side *between* them are equal to the corresponding parts of the other.
4. **AAS** (angle-angle-side): two angles and a side *not* between them are equal to the corresponding parts of the other.



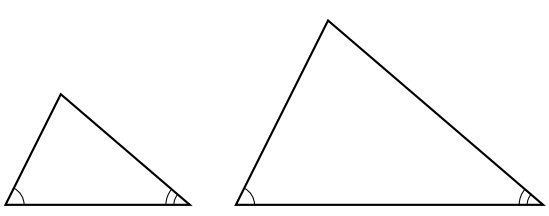
1. SSS 2. SAS 3. ASA 4. AAS

AAS is really the same as ASA: if two angles of the triangles are equal, then the third angles are also equal, because the angles of every triangle add up to 180°.

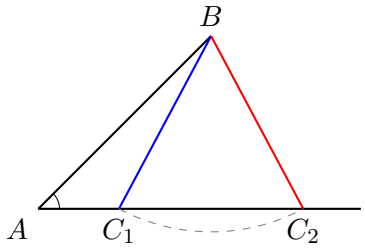
What Is Not Enough

AAA. If all three angles are equal, the triangles have the same shape, but they can have different sizes. Such triangles are called *similar*, but they are not necessarily congruent. (See the picture on the left below.)

SSA. Two sides and an angle that is *not* between them are not enough either. In the picture on the right, the triangles ABC_1 and ABC_2 have the same angle at A, the same side AB, and the same side $BC_1 = BC_2$, but they are clearly not congruent. The side starting at B can swing to two different positions.



AAA: similar, but not congruent



SSA: two different triangles

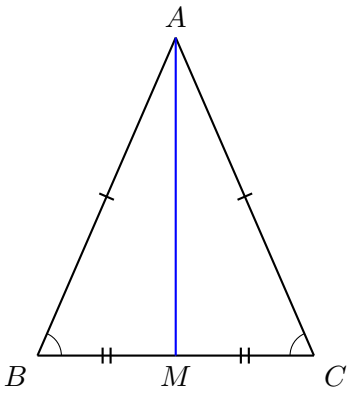
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Using Congruent Triangles

Congruent triangles are useful because once we know that two triangles are congruent, we know that *all* of their corresponding parts are equal, even the ones we did not measure.

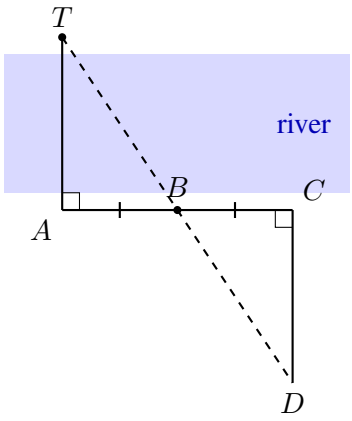
Example 1. Triangle ABC is isosceles: $AB = AC$. Show that its base angles are equal: $\angle B = \angle C$.

Solution. Let M be the midpoint of side BC, and draw the segment AM. This divides the triangle into two smaller triangles, ABM and ACM.



The two triangles have three pairs of equal sides: $AB = AC$ (given), $BM = CM$ (because M is the midpoint), and $AM = AM$ (it is a common side). So by SSS, $\triangle ABM \cong \triangle ACM$. Corresponding angles of congruent triangles are equal, so $\angle B = \angle C$.

Example 2. A hiker wants to know how wide a river is, but she cannot cross it. There is a tree at point T on the other side. She stands at point A, exactly across from the tree. Then she walks along the river bank to point B and places a stick there. She keeps walking the same distance to point C, so that $AB = BC$. Finally, she turns away from the river at a right angle and walks until she sees the stick and the tree in one line. She stops at this point, D. How wide is the river?



Solution. Compare the triangles ABT and CBD. The angles at A and at C are both right angles. The sides between these angles and B are equal: $AB = BC$. The angles at B are also equal, because they are vertical angles. So by ASA, $\triangle ABT \cong \triangle CBD$. In particular, $AT = CD$. The width of the river, AT, is the same as the distance CD, which the hiker can simply measure on her side of the river.

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Practice Problems

1. We know that $\triangle ABC \cong \triangle PQR$.
   a) List all pairs of corresponding sides and corresponding angles.
   b) If $AB = 7$ cm, $\angle Q = 40°$, and $\angle C = 65°$, find PQ, $\angle B$, $\angle R$, and $\angle P$.

2. In each case, we know some parts of two triangles. Decide whether the two triangles must be congruent. If they must, name the criterion.
   a) All three sides are 5, 7, and 9 cm in both triangles.
   b) Two sides are 6 and 8 cm in both triangles, and the angle between them is $50°$ in both.
   c) The angles are $30°$, $70°$, and $80°$ in both triangles.
   d) Two angles are $40°$ and $60°$ in both triangles, and the side between them is 10 cm in both.
   e) Two angles are $40°$ and $60°$ in both triangles, and the side opposite the $40°$ angle is 7 cm in both.
   f) Two sides are 8 and 6 cm in both triangles, and the angle opposite the 6 cm side is $30°$ in both.

3. In the quadrilateral $ABCD$, opposite sides are equal: $AB = CD$ and $BC = DA$. The diagonal AC divides the quadrilateral into two triangles. Explain why these two triangles are congruent, and why $\angle B = \angle D$.

4. The point M is the midpoint of the segment AB. A line through M is perpendicular to AB, and P is any point on this line. Use congruent triangles to explain why $PA = PB$.

5. In Example 2, the hiker made sure that $AB = BC$. Explain why the method would not work if she walked a different distance from B to C than from A to B.

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Answers

1. a) $AB = PQ,\ BC = QR,\ CA = RP,\ \angle A = \angle P,\ \angle B = \angle Q,\ \angle C = \angle R$

   b) $PQ = AB = 7$ cm, $\angle B = \angle Q = 40°,\ \angle R = \angle C = 65°,\ \angle P = 180° - 40° - 65° = 75°$

2. a) congruent, SSS b) congruent, SAS c) not necessarily: the triangles are similar, but they may have different sizes

   d) congruent, ASA e) congruent, AAS f) not necessarily: this is SSA, and there can be two different triangles

3. The triangles ABC and CDA have three pairs of equal sides: $AB = CD$, $BC = DA$, and the common side AC. So by SSS, $\triangle ABC \cong \triangle CDA$. The angle B is opposite the side AC in the first triangle, and the angle D is opposite the same side in the second triangle, so they are corresponding angles, and $\angle B = \angle D$.

4. The triangles AMP and BMP have $AM = BM$ (because M is the midpoint), a right angle at M in both, and the common side MP. So by SAS, $\triangle AMP \cong \triangle BMP$, and therefore $PA = PB$.

5. The ASA argument needs the side between the two angles to be equal in both triangles, that is, $AB = BC$. If the two distances were different, the triangles ABT and CBD would still have the same angles, but they would have different sizes. Then CD would not be equal to AT.

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