

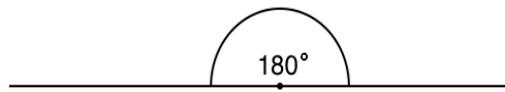
In mathematics, we will be dealing with different types of true statements. Some examples for these are definitions, axioms and theorems. What are these?

A **definition** is a type of statement in which we agree how we will refer to things. It is true in a sense because it just sets an agreement about labeling things.

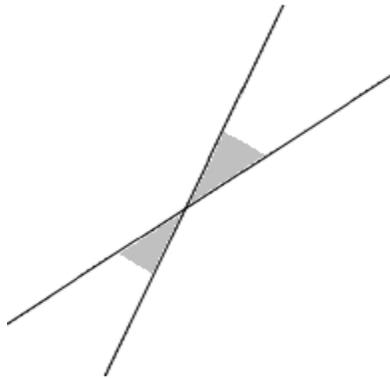
An **axiom** is a statement that we accept as true, without requiring proof of it. It usually agrees with our natural instincts and they "feel true". One example is the statement: "Two points uniquely determine a straight line".

A **theorem** is a statement that we prove to be true. But what does it mean to prove something? It means to derive it from the axioms. Mathematicians set down a set of basic 'truths', the axioms. Everything we prove, we derive them from the axioms. The following proof is a presentation of this. We will use two axioms to prove a theorem. Here is all we need:

**Definition:** The angle shown on the picture below is called the straight angle and it measures  $180^\circ$ .

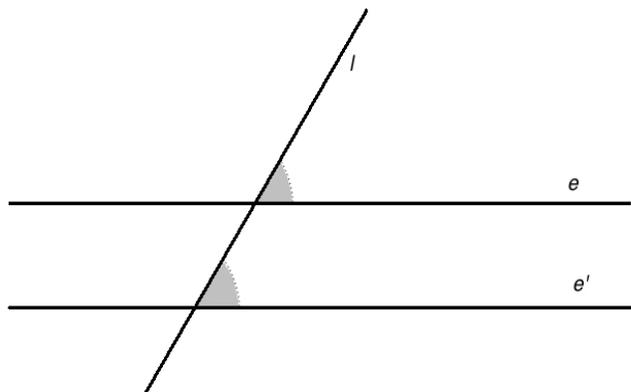


**Axiom 1:** If two straight lines intersect each other, then the opposite angles formed have equal measures as the picture below shows.



These angles are called vertical angles.

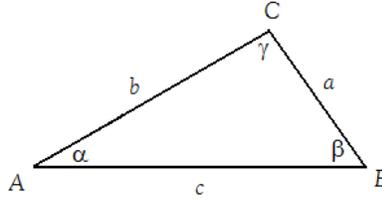
**Axiom 2:** If  $e$  and  $e'$  are parallel lines, and  $l$  is a line intersecting these lines, then the two angles marked on the picture below have equal measures. We say that line  $l$  is a transversal for the lines  $e$  and  $e'$ .



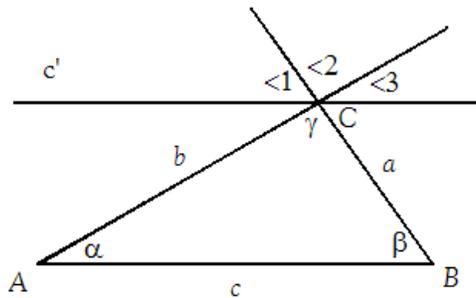
We are now ready to state and prove our first theorem.

**Theorem:** If  $\alpha$ ,  $\beta$ , and  $\gamma$  are the three angles of a triangle, then  $\alpha + \beta + \gamma = 180^\circ$ .

**Proof.** Let  $ABC$  be any triangle. Let us denote the angles by  $\alpha$ ,  $\beta$ , and  $\gamma$ , and the sides by  $a$ ,  $b$ , and  $c$  as shown on the picture below.



Let us draw the lines  $a$ ,  $b$ , and  $c$  longer, beyond the triangle. As often times in geometry, proofs are based on a smartly drawn, single line. In this case, the 'magical line' that will give us the proof, is a line, we will call it  $c'$ , that is parallel to  $c$  and passes through the point  $C$ . There are three new angles formed, we will label them as  $\angle 1$ ,  $\angle 2$ , and  $\angle 3$ , as shown on the picture below.



Because of Axiom 1,  $\gamma = \angle 2$ . Those two angles are vertical as the lines  $a$  and  $b$  intersect. Because  $a$  is a transversal for the parallel lines  $c$  and  $c'$ ,  $\angle 1 = \beta$ . Because  $b$  is a transversal for the parallel lines  $c$  and  $c'$ ,  $\angle 3 = \alpha$ . We can observe that  $\angle 1 + \angle 2 + \angle 3 = 180^\circ$ , since they are forming a straight angle together. So we have:

$$\underbrace{\angle 1}_{\beta} + \underbrace{\angle 2}_{\gamma} + \underbrace{\angle 3}_{\alpha} = 180^\circ$$

$$\alpha + \beta + \gamma = 180^\circ$$

This concludes our proof. ■

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