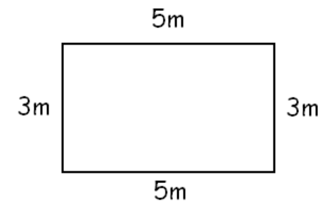
Part 1 - Perimeter

Definition: The **perimeter** of any geometric object is the length of its boundary.

In general, we can always think of the perimeter as a fencing problem. If we have a property, how long of a fence do we need to buy to completely fence around the property? We will denote perimeter by P. Perimeter is a *length*, we measure it in meters (m), centimeters (cm), inches (in), feet (ft), kilometers (km), or miles (mi).

Example 1. Compute the perimeter of a rectangle with sides 3 meters and 5 meters long.



Solution: If we think fencing, we mentally walk around a rectangle-shaped property to figure out how much fencing to buy. That is the same as simply adding the lengths of all four sides sides. The lengths of only two sides were given, but this should not be a problem. The opposite sides of a rectangle are equally long. Thus we can compute the perimeter as

$$P = 3\,\mathrm{m} + 5\,\mathrm{m} + 3\,\mathrm{m} + 5\,\mathrm{m} = 16\,\mathrm{m}$$

So the perimeter of this rectangle is $\boxed{P = 16\,\mathrm{m}}$.

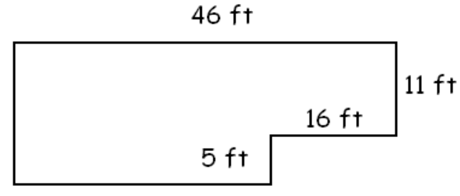
In general, the perimeter of a quadrilateral with sides a, b, c, and d is $P = a + b + c + d$. In case of a rectangle, the opposite sides are equally long, so $c = a$ and $d = b$ and this makes the primeter formula simpler.

Theorem: The perimeter of a rectangle with sides a and b is $P = 2a + 2b$.

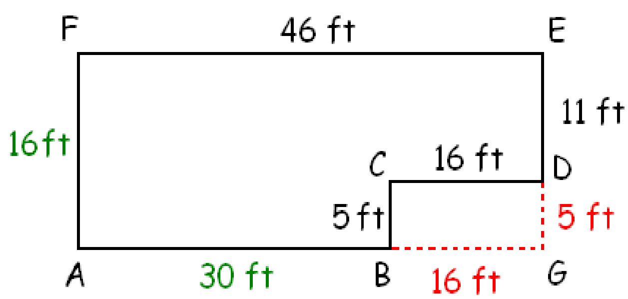
This means that in our previous example, the perimeter of a $3\,\mathrm{m}$ by $5\,\mathrm{m}$ rectangle is

$$P = 2\,(3\,\mathrm{m}) + 2\,(5\,\mathrm{m}) = 6\,\mathrm{m} + 10\,\mathrm{m} = 16\,\mathrm{m}$$

Example 2. Find the perimeter of the object shown. Angles that look like right angles are right angles.



Solution: It is much easier to discuss geometry if we label points and sides. Consider the picture shown. We are clearly missing the lengths of some sides for the perimeter, so we need to figure out those lengths first.



We first draw line AB beyond point B and line ED beyond point D as shown. These lines intersect each other in point G. Since $BGDC$ is a rectangle, its opposite sides are equally long. Therefore, $BG = 16\,\mathrm{ft}$ and $DG = 5\,\mathrm{ft}$.

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The quadrilateral $AGEF$ is also a rectangle, and so FA is the same length as EG.
$FA = 16\,\text{ft}$, because $FA = EG = ED + DG = 11\,\text{ft} + 5\,\text{ft} = 16\,\text{ft}$. Also, AG is as long as FE.

$$\begin{aligned} AG &= FE \\ AB + BG &= 46\,\text{ft} && \text{we know } BG = 16\,\text{ft} \\ AB + 16\,\text{ft} &= 46\,\text{ft} && \text{subtract } 16\,\text{ft} \\ AB &= 30\,\text{ft} \end{aligned}$$

We can now compute the perimeter, starting from point A, and moving counterclockwise.

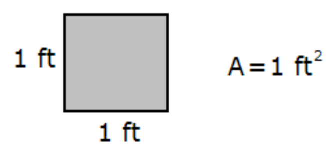
$$P = AB + BC + CD + DE + EF + FA = 30\,\text{ft} + 5\,\text{ft} + 16\,\text{ft} + 11\,\text{ft} + 46\,\text{ft} + 16\,\text{ft} = \boxed{124\,\text{ft}}$$

Part 2 - Area

The **area** of a geometric object is a measurement of its surface.

Understanding and remembering area formulas are easier if we know how they were derived. While we could think about perimeter as a fencing problem, area can be thought of as follows. Suppose a geometric object is a room. How many tiles do we need to buy to cover the entire room? Of course, we have to first agree on the size of the tiles we use to measure area.

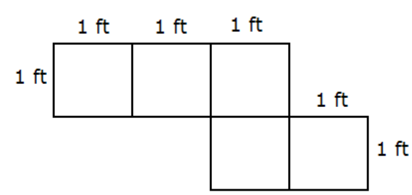
Definition: The **area** of a $1\,\text{ft}$ by $1\,\text{ft}$ square (shown on the picture) is defined to be $1\,\text{ft}^2$ (square-foot). Similar definitions can be formulated with mi^2, cm^2, in^2, etc. The area of any object, measured in ft^2, is the number of these $1\,\text{ft}$ by $1\,\text{ft}$ square tiles needed to cover the object, cutting and pasting allowed.



Area is not a length like perimeter. Area is measured in square-meters (m^2), square-centimeters (cm^2), square-inches (in^2), square-feet (ft^2), square-kilometers (km^2), or square-miles (mi^2). etc., and is usually denoted by A.

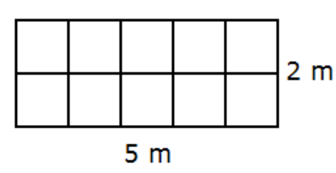
Example 3. Find the area of the figure shown on the picture.

Solution: We simply count the tiles we need to cover this object. Since the figure can be covered using five unit tiles, its area is $\boxed{A = 5\,\text{ft}^2}$.



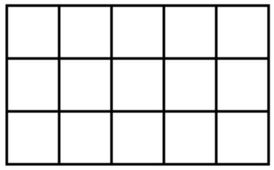
Example 4. Find the area of the figure shown on the picture.

Solution: Since the figure can be covered using ten $1\,\text{m}$ by $1\,\text{m}$ squares, its area is $\boxed{A = 10\,\text{m}^2}$.



Theorem: The area of a rectangle with sides x and y is $A = xy$.

Proof: We will not formally prove this theorem. Instead, we will just informally argue for this formula. The main idea should be clear from the previous example. Consider a rectangle with sides $3\,\mathrm{m}$ and $5\,\mathrm{m}$. The area of this rectangle will be as many square-meters as many $1\,\mathrm{m}$ by $1\,\mathrm{m}$ square tiles are needed to cover it.



Once we place a grid on the rectangle, it is easy to see how many such squares are needed. The rectangle is composed of five columns of squares, where each column consists of three squares. Thus we split the rectangle into fifteen unit tiles, and so the area is $15\,\mathrm{m}^2$. This shows that as long as the lengths of the sides are integers, we can place a grid on it, and the number of unit square tiles is the product of the length of the two sides.

In reality, this theorem is very difficult to prove, because not all side lengths happen to be integers. Mathematicians proved that this formula is true even if the sides of the rectangle are not integers.

Another interesting fact is that logically, we counted how many meter^2 we have. The computation for the area, however, is slightly different with regards to units. Instead of counting meter^2, we literally multiply meter by meter.

$$A = xy = 3\,\mathrm{m}\,(5\,\mathrm{m}) = 15\,\mathrm{m}^2 \quad \text{and not} \quad 15 \cdot 1\,\mathrm{m}^2$$

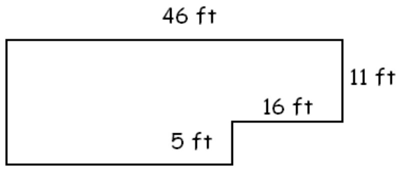
Area computation will always yield for the right unit.

Example 5. Find the area of a rectangle with sides 13 in and 7 in.

Solution: We apply the formula.

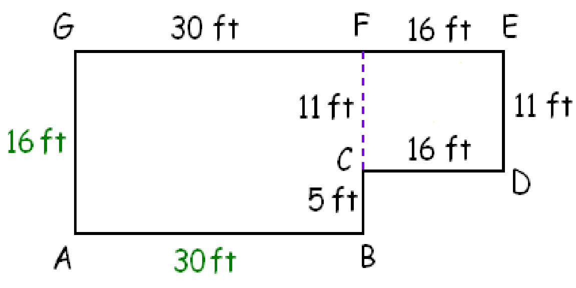
$$A = xy = 13\,\mathrm{in}\,(7\,\mathrm{in}) = \boxed{91\,\mathrm{in}^2}$$

Example 6. Find the area of the object shown. Angles that look like right angles are right angles.



Solution: To compute the area, we have two options.

Method 1: We can think of the region as the sum of two rectangles: $CDEF$ is a rectangle with sides $11\,\mathrm{ft}$ by $16\,\mathrm{ft}$, and $ABFG$ is a rectangle with sides $30\,\mathrm{ft}$ by $16\,\mathrm{ft}$. So the area is



$$\begin{aligned} A &= A_{CDEF} + A_{ABFG} \\ &= 11\,\mathrm{ft} \cdot 16\,\mathrm{ft} + 16\,\mathrm{ft} \cdot 30\,\mathrm{ft} \\ &= 176\,\mathrm{ft}^2 + 480\,\mathrm{ft}^2 = \boxed{656\,\mathrm{ft}^2} \end{aligned}$$

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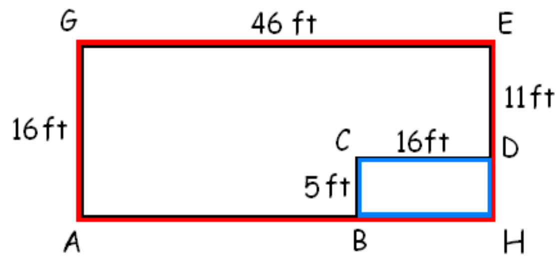
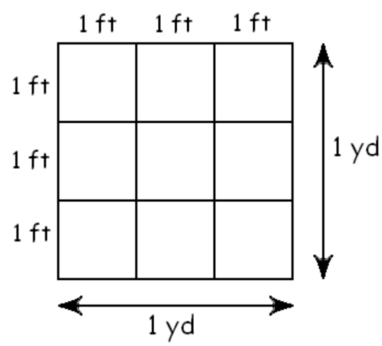
Method 2: We can also think of the region as a big rectangle from which a corner was removed.

The big (red) rectangle, $AHEG$ has sides $16\,\mathrm{ft}$ and $46\,\mathrm{ft}$ long. The smaller (blue) rectangle has sides $5\,\mathrm{ft}$ by $16\,\mathrm{ft}$. So the area is the difference.



$$\begin{aligned} A &= A_{AHEG} - A_{BHDC} \\ &= 46\,\mathrm{ft} \cdot 16\,\mathrm{ft} - 16\,\mathrm{ft} \cdot 5\,\mathrm{ft} \\ &= 736\,\mathrm{ft}^2 - 80\,\mathrm{ft}^2 = \boxed{656\,\mathrm{ft}^2} \end{aligned}$$

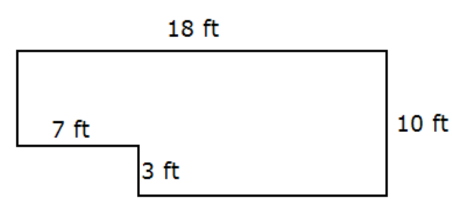
Discussion: One yard equals to three feet. Consider the picture below and discuss: how many square-feet is one square-yard? Can you show the same result algebraically?



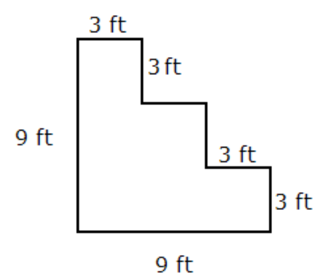
Practice Problems

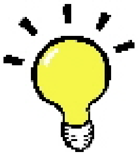
1. Compute the perimeter and area of a rectangle with sides $12\,\mathrm{cm}$ by $8\,\mathrm{cm}$.

2. Consider the figure shown. Angles that look like right angles are right anges.
   a) Compute the perimeter of the figure. Include units in your answer.
   b) Compute the area of the figure. Include units in your answer.



3. Consider the figure shown. Angles that look like right angles are right anges.
   a) Compute the perimeter of the figure. Include units in your answer.
   b) Compute the area of the figure. Include units in your answer.





Answers

Discussion

$1\,\text{yd}^2 = 9\,\text{ft}^2$

Algebraically: if $1\,\text{yd} = 3\,\text{ft}$, then $1\,\text{yd}^2 = 1\,\text{yd} \cdot 1\,\text{yd} = 3\,\text{ft} \cdot 3\,\text{ft} = 9\,\text{ft}^2$

Practice Problems

1. $P = 40\,\text{cm}$, $A = 96\,\text{cm}^2$
2. $P = 56\,\text{ft}$, $A = 159\,\text{ft}^2$
3. $P = 36\,\text{ft}$, $A = 54\,\text{ft}^2$

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