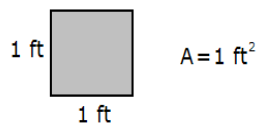


Area of Rectangles and Right Triangles

The **area** of a geometric object is a measurement of its surface.

Understanding and remembering area formulas are easier if we know how they were derived. While we could think about perimeter as a fencing problem, area can be thought of as follows. Suppose a geometric object is a room. How many tiles do we need to buy to cover the entire room? Of course, we have to first agree on the size of the tiles we use to measure area.

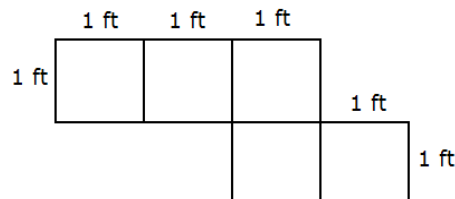
Definition: The **area** of a 1 ft by 1 ft square (shown on the picture below) is defined to be 1 ft^2 . (Similar definitions can be formulated with mi^2 , cm^2 , in^2 , etc.)



The area of any object, measured in ft^2 , is the number of 1 ft by 1 ft square needed to cover the object, cutting and pasting allowed.

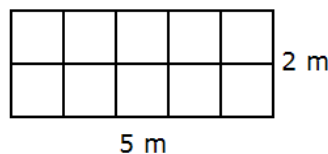
Area is not a length like perimeter. Area is always measured in ft^2 , mi^2 , cm^2 , in^2 , etc., and is usually denoted by A .

Example 1: Find the area of the figure shown on the picture below.



Solution: We simply count the tiles we need to cover this object. Since the figure can be covered by 5 1 ft by 1 ft squares, its area is $A = 5 \text{ ft}^2$.

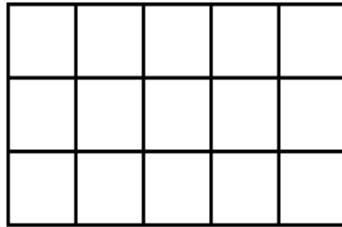
Example 2: Find the area of the figure shown on the picture below.



Solution: Since the figure can be covered using 10 1 m by 1 m squares, its area is $A = 10 \text{ m}^2$.

Theorem: The area of a rectangle with sides x and y is $A = xy$.

Proof. Consider a rectangle with sides 3 m and 5 m. The area of this rectangle will be as many m^2 as many 1 m by 1 m square is needed to cover it. Once we place this grid on the rectangle, it is easy to see, just how many squares we need.



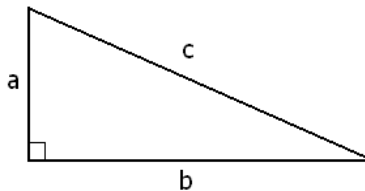
We used exactly 15 squares to cover the rectangle, and so the area is 15 m^2 .

Mathematicians also proved that this formula is true even if the sides of the rectangle are not integers. It is interesting to see that we basically counted how many meter^2 we have. A computation for the area, including the units is slightly different. Instead of counting meter^2 , we literally multiply meter by meter.

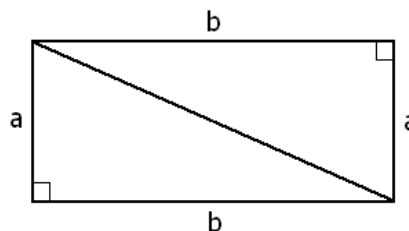
$$A = xy = 3 \text{ m} (5 \text{ m}) = 15 \text{ m}^2$$

Area computation will always yield for the right unit.

Theorem: The area of a right triangle with sides a , b , and c (where c is the longest side, also called the hypotenuse) is $A = \frac{ab}{2}$.



Proof: It is very easy to see that every right triangle is basically half of a rectangle. We can make a rectangle if we use two identical right triangles as shown on the picture below.



Since the rectangle's sides are a and b , its area is ab . The area of our right triangle must be half of it. Thus

$$A = \frac{ab}{2}$$

Notice that we never used the length of the longest side, c .

1. Find the area of a rectangle with sides 13 in and 7 in.

Solution: We apply the formula.

$$A = xy = 13 \text{ in} (7 \text{ in}) = 91 \text{ in}^2$$

2. Find the area of the right triangle with sides 5 m, 12 m, and 13 m long.

Solution: It is important to know that the largest side, 13 m long, is not needed for this computation. With labeling $a = 5 \text{ m}$ and $b = 12 \text{ m}$, the area is

$$A = \frac{ab}{2} = \frac{5 \text{ m} (12 \text{ m})}{2} = 30 \text{ m}^2$$