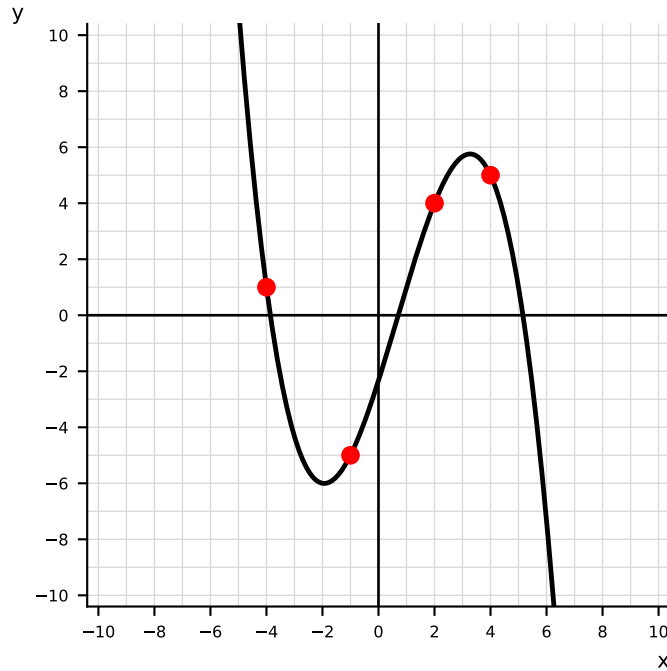


Definition: The graph of an equation in x and y is the set of all points $P(x, y)$ whose coordinates are solution of the equation.

1. Consider the graph shown below.



Note that the points $A(-4, 1)$, $B(2, 4)$, $C(4, 5)$, and $D(-1, -5)$ are on the graph.

Determine which of the following equations belongs to this graph.

- a) $8y = 5x^2 + 14x - 16$
- b) $2y - 3 = x + 3$
- c) $6(1 - y) = (x - 1)(x^2 - x - 20)$

- (a) Consider the equation $8y = 5x^2 + 14x - 16$ and points $A(-4, 1)$, $B(2, 4)$, and $C(4, 5)$.

- i. Is the point $A(-4, 1)$ on the graph of $8y = 5x^2 + 14x - 16$?

Solution: We need to determine whether the coordinates of A are a solution of the equation or not. Let $x = -4$ and $y = 1$. Is this pair a solution of $8y = 5x^2 + 14x - 16$?

$$\begin{aligned}\text{LHS} &= 8(1) = 8 \\ \text{RHS} &= 5(-4)^2 + 14(-4) - 16 = 5 \cdot 16 + 14(-4) - 16 = 80 - 56 - 16 \\ &= 24 - 16 = 8\end{aligned}$$

Since $\text{LHS} = \text{RHS}$, the point $A(-4, 1)$ is on the graph of $8y = 5x^2 + 14x - 16$.

- ii. Is the point $A(2, 4)$ on the graph of $8y = 5x^2 + 14x - 16$?

Solution: We need to determine whether the coordinates of A are a solution of the equation or not. Let $x = 2$ and $y = 4$. Is this pair a solution of $8y = 5x^2 + 14x - 16$?

$$\begin{aligned}\text{LHS} &= 8(4) = 32 \\ \text{RHS} &= 5(2)^2 + 14(2) - 16 = 5 \cdot 4 + 14 \cdot 2 - 16 = 20 + 28 - 16 \\ &= 48 - 16 = 32\end{aligned}$$

Since $\text{LHS} = \text{RHS}$, the point $B(2, 4)$ is on the graph of $8y = 5x^2 + 14x - 16$.

- iii. Is the point $C(4, 5)$ on the graph of $8y = 5x^2 + 14x - 16$?

Solution: We need to determine whether the coordinates of A are a solution of the equation or not. Let $x = 4$ and $y = 5$. Is this pair a solution of $8y = 5x^2 + 14x - 16$?

$$\begin{aligned}\text{LHS} &= 8(5) = 40 \\ \text{RHS} &= 5(4)^2 + 14(4) - 16 = 5 \cdot 16 + 14 \cdot 4 - 16 = 80 + 56 - 16 \\ &= 136 - 16 = 120\end{aligned}$$

Since $\text{LHS} \neq \text{RHS}$, the point $C(4, 5)$ is NOT on the graph of $8y = 5x^2 + 14x - 16$.

- iv. Is it possible that the graph is that of $8y = 5x^2 + 14x - 16$?

No, since C is not on the graph of the equation of $8y = 5x^2 + 14x - 16$.

- (b) Consider the equation $2y - 3 = x + 3$.

- i. Is the point $A(-4, 1)$ on the graph of $2y - 3 = x + 3$?

Solution: We need to determine whether the coordinates of A are a solution of the equation or not. Let $x = -4$ and $y = 1$. Is this pair a solution of $2y - 3 = x + 3$?

$$\begin{aligned}\text{LHS} &= 2(1) - 3 = 2 - 3 = -1 \\ \text{RHS} &= -4 + 3 = -1\end{aligned}$$

Since $\text{LHS} = \text{RHS}$, the point $A(-4, 1)$ is on the graph of $2y - 3 = x + 3$.

- ii. Is the point $A(2, 4)$ on the graph of $2y - 3 = x + 3$?

Solution: We need to determine whether the coordinates of A are a solution of the equation or not. Let $x = 2$ and $y = 4$. Is this pair a solution of $2y - 3 = x + 3$?

$$\begin{aligned}\text{LHS} &= 2(4) - 3 = 8 - 3 = 5 \\ \text{RHS} &= 2 + 3 = 5\end{aligned}$$

Since $\text{LHS} = \text{RHS}$, the point $B(2, 4)$ is on the graph of $2y - 3 = x + 3$.

- iii. Is the point $C(4, 5)$ on the graph of $2y - 3 = x + 3$?

Solution: We need to determine whether the coordinates of A are a solution of the equation or not. Let $x = 4$ and $y = 5$. Is this pair a solution of $2y - 3 = x + 3$?

$$\text{LHS} = 2(5) - 3 = 10 - 3 = 7$$

$$\text{RHS} = 4 + 3 = 7$$

Since $\text{LHS} = \text{RHS}$, the point $C(4, 5)$ is on the graph of $2y - 3 = x + 3$.

- iv. Is it possible that the graph is that of $2y - 3 = x + 3$?

Based on the points A , B , and C , the answer is yes, since A , B , and C are all on the graph of the equation $2y - 3 = x + 3$.

- (c) Consider the equation $6(1 - y) = (x - 1)(x^2 - x - 20)$.

- i. Is the point $A(-4, 1)$ on the graph of $6(1 - y) = (x - 1)(x^2 - x - 20)$?

Solution: We need to determine whether the coordinates of A are a solution of the equation or not. Let $x = -4$ and $y = 1$. Is this pair a solution of $6(1 - y) = (x - 1)(x^2 - x - 20)$?

$$\text{LHS} = 6(1 - 1) = 6(0) = 0$$

$$\begin{aligned} \text{RHS} &= ((-4) - 1)((-4)^2 - (-4) - 20) = -5(16 + 4 - 20) = -5(20 - 20) \\ &= -5 \cdot 0 = 0 \end{aligned}$$

Since $\text{LHS} = \text{RHS}$, the point $A(-4, 1)$ is on the graph of $6(1 - y) = (x - 1)(x^2 - x - 20)$.

- ii. Is the point $A(2, 4)$ on the graph of $6(1 - y) = (x - 1)(x^2 - x - 20)$?

Solution: We need to determine whether the coordinates of A are a solution of the equation or not. Let $x = 2$ and $y = 4$. Is this pair a solution of $6(1 - y) = (x - 1)(x^2 - x - 20)$?

$$\text{LHS} = 6(1 - 4) = 6(-3) = -18$$

$$\begin{aligned} \text{RHS} &= (2 - 1)(2^2 - 2 - 20) = 1(4 - 2 - 20) = 1(2 - 20) \\ &= 1(-18) = -18 \end{aligned}$$

Since $\text{LHS} = \text{RHS}$, the point $B(2, 4)$ is on the graph of $6(1 - y) = (x - 1)(x^2 - x - 20)$.

- iii. Is the point $C(4, 5)$ on the graph of $6(1 - y) = (x - 1)(x^2 - x - 20)$?

Solution: We need to determine whether the coordinates of A are a solution of the equation or not. Let $x = 4$ and $y = 5$. Is this pair a solution of $6(1 - y) = (x - 1)(x^2 - x - 20)$?

$$\text{LHS} = 6(1 - 5) = 6(-4) = -24$$

$$\begin{aligned} \text{RHS} &= (4 - 1)(4^2 - 4 - 20) = 3(16 - 4 - 20) = 3(12 - 20) \\ &= 3(-8) = -24 \end{aligned}$$

Since $\text{LHS} = \text{RHS}$, the point $C(4, 5)$ is on the graph of $6(1 - y) = (x - 1)(x^2 - x - 20)$.

- iv. Is it possible that the graph is that of $6(1 - y) = (x - 1)(x^2 - x - 20)$?

Based on the points A , B , and C , the answer is yes, since A , B , and C are all on the graph of the equation $6(1 - y) = (x - 1)(x^2 - x - 20)$.

We need to rule out one of the equations from b) and c). Let us test these equations using point $D(-1, -5)$.

For part b):

Is the point $D(-1, -5)$ on the graph of $2y - 3 = x + 3$?

Solution: We need to determine whether the coordinates of A are a solution of the equation or not. Let $x = -1$ and $y = -5$. Is this pair a solution of $2y - 3 = x + 3$?

$$\begin{aligned}\text{LHS} &= 2(-5) - 3 = -10 - 3 = -13 \\ \text{RHS} &= -1 + 3 = 2\end{aligned}$$

Since $\text{LHS} \neq \text{RHS}$, the point $D(-1, -5)$ is NOT on the graph of $2y - 3 = x + 3$. Consequently, the graph is NOT that of $2y - 3 = x + 3$.

For part c):

Is the point $D(-1, -5)$ on the graph of $6(1 - y) = (x - 1)(x^2 - x - 20)$?

Solution: We need to determine whether the coordinates of A are a solution of the equation or not. Let $x = -1$ and $y = -5$. Is this pair a solution of $6(1 - y) = (x - 1)(x^2 - x - 20)$?

$$\begin{aligned}\text{LHS} &= 6(1 - (-5)) = 6 \cdot 6 = 36 \\ \text{RHS} &= (-1 - 1)((-1)^2 - (-1) - 20) = -2(1 + 1 - 20) = -2(2 - 20) \\ &= -2(-18) = 36\end{aligned}$$

Since $\text{LHS} = \text{RHS}$, the point $D(-1, -5)$ is on the graph of $6(1 - y) = (x - 1)(x^2 - x - 20)$. Consequently, it is possible that the graph is that of $6(1 - y) = (x - 1)(x^2 - x - 20)$.