Equations that are in x, or in y, or in x and y can be graphed. **The graph of such an equation is the set of all points $P(x, y)$ for which the coordinates x and y form a solution of the equation.**

In case of linear (of degree one) equations, the graph is a straight line. There are several forms of a line's equation. Two of them are as follows (there are more).

$$\begin{aligned} y &= mx + b && \text{slope-intercept form} \\ Ax + By &= C && \text{general form} \end{aligned}$$

In this section, we will present two methods of graphing a line.

Method 1 - Graphing by finding points

1. Graph the line $y = -2x + 3$

Solution: We will find points on this line and connect the dots. Since the graph is a straight line, theoretically it doesn't matter which of its many points we will find. To safeguard against computational errors and to guarantee precision, at least four or five points should be plotted.

Here is how we can find a point.

Step 1. Let us freely choose any value for x. We will go with $x = 4$. We will look for a point on this line with $x-$coordinate 4.

Step 2. To find the $y-$coordinate of this point, we will use the equation of the line that establishes a connection between the $x-$ and $y-$coordinates of the points.

$$\begin{aligned} y &= \ ? \text{ if } x = 4 \\ x &= 4 \text{ and } y = -2x + 3 \implies y = -2(4) + 3 = -8 + 3 = -5 \end{aligned}$$

If $x = 4$, then $y = -5$. Thus we found the point $(4, -5)$ that is on this line. We repeat the process with other values for x to find other points on the line.

Let $x = 0$. We will compute the value of y.

$$\begin{aligned} y &= \ ? \text{ if } x = 0 \\ x &= 0 \text{ and } y = -2x + 3 \implies y = -2(0) + 3 = 0 + 3 = 3 \implies (0, 3) \end{aligned}$$

If $x = 0$, then $y = 3$. Thus we found the point $(0, 3)$ that is on this line.

Let $x = -2$. We will compute the value of y.

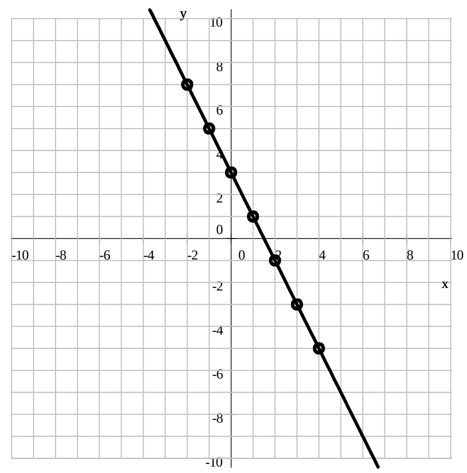
$$\begin{aligned} y &= \ ? \text{ if } x = -2 \\ x &= -2 \text{ and } y = -2x + 3 \implies y = -2(-2) + 3 = 4 + 3 = 7 \implies (-2, 7) \end{aligned}$$

If $x = -2$, then $y = 7$. Thus we found the point $(-2, 7)$ that is on this line.

We continue to find additional points in this manner. We organize the results in a table:

x	y	$\Longrightarrow$	$P(x, y)$
-2	7		$(-2, 7)$
-1	5		$(-1, 5)$
0	3		$(0, 3)$
1	1		$(1, 1)$
2	-1		$(2, -1)$
3	-3		$(3, -3)$
4	-5		$(4, -5)$

We plot these points and connect the dots.



The point where the graph intersects the $x-$axis is called the $x-$intercept. The point where the graph intersects the $y-$axis is called the $y-$intercept. This line's $y-$intercept is $(0, 3)$.

2. Graph the line $y = \frac{1}{2}x - 1$

Solution: We freely select any value for x. We find the $y-$coordinate of the point using the equation of the line.

Let $x = -4$. We will compute the value of y.

$$\begin{aligned} y &= \ ? \text{ if } x = -4 \\ x &= 4 \text{ and } y = \frac{1}{2}x - 1 \quad \Longrightarrow \quad y = \frac{1}{2}(-4) - 1 = -2 - 1 = -3 \end{aligned}$$

If $x = -4$, then $y = -3$. Thus we found the point $(-4, -3)$ on this line. We repeat the process with other values for x to find other points on the line.

Let $x = -3$. We will compute the value of y.

$$\begin{aligned} y &= \ ? \text{ if } x = -3 \\ x &= -3 \text{ and } y = \frac{1}{2}x - 1 \quad \Longrightarrow \quad y = \frac{1}{2}(-3) - 1 = -\frac{3}{2} - 1 = -\frac{5}{2} \end{aligned}$$

If $x = -3$, then $y = -\frac{5}{2}$. Thus we found the point $\left(-3, -\frac{5}{2}\right)$ on this line. Although the point we found is correct, its $y-$coordinate is not an integer. This makes the plotting of this point more difficult and also inaccurate.

Whenever possible, we should rely on graphing lattice. points. **A lattice point is a point of whose both coordinates are integers.**

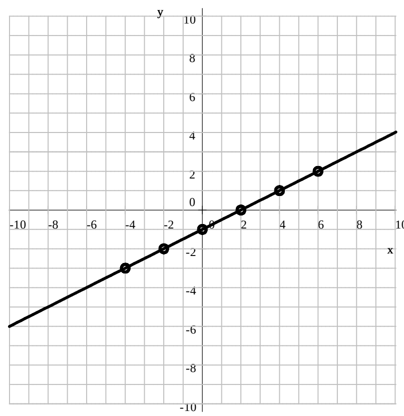
Let $x = -2$. We will compute the value of y.

$$\begin{aligned} y &= \ ? \text{ if } x = -2 \\ x &= -2 \text{ and } y = \frac{1}{2}x - 1 \implies y = \frac{1}{2}(-2) - 1 = -1 - 1 = -2 \end{aligned}$$

If $x = -2$, then $y = -2$. Thus we found the point $(-2, -2)$ on this line.

We continue to find additional points in this manner. Notice that we get lattice points if we use even numbers for x. We organize the results in a table, and then plot these points and connect the dots.

x	y	$\Longrightarrow$	$P(x,y)$
-4	-3		$(-4,-3)$
-3	$-\frac{5}{2}$		$\left(-3,-\frac{5}{2}\right)$
-2	-2		$(-2,-2)$
-1	$-\frac{3}{2}$		$\left(-1,-\frac{3}{2}\right)$
0	-1		$(0,-1)$
1	$-\frac{1}{2}$		$\left(1,-\frac{1}{2}\right)$
2	0		$(2,0)$
4	1		$(4,1)$
6	2		$(6,2)$



The point where the graph intersects the $x-$axis is called the $x-$intercept. The point where the graph intersects the $y-$axis is called the $y-$intercept. This line's $x-$intercept is $(2,0)$ and $y-$intercept is $(0,-1)$.

3. Graph the line $2x + 3y = -12$

Solution: We freely select any value for x. We find the $y-$coordinate of the point using the equation of the line. We substitute the value for x and solve the equation for y.
Let $x = 0$. We will compute the value of y.

$$\begin{aligned} y &= \ ? \text{ if } x = 0 \\ x &= 0 \text{ and } 2x + 3y = -12 \implies 2(0) + 3y = -12 \end{aligned}$$

$$\begin{aligned} 2(0) + 3y &= -12 && \text{solve for } y \\ 3y &= -12 && \text{divide by 3} \\ y &= -4 \end{aligned}$$

If $x = 0$, then $y = -4$. Thus we found the point $(0, -4)$ on this line. We repeat the process with other values for x to find other points on the line.

Let $x = 2$. We will compute the value of y.

$$\begin{aligned} y &= \ ? \text{ if } x = 2 \\ x &= 2 \text{ and } 2x + 3y = -12 \implies 2(2) + 3y = -12 \end{aligned}$$

$$\begin{aligned} 2(2)+3y &= -12 && \text{solve for } y \\ 3y+4 &= -12 && \text{subtract } 4 \\ 3y &= -16 && \text{divide by } 3 \\ y &= -\frac{16}{3} \end{aligned}$$

If $x = 2$, then $y = -\frac{16}{3}$. Thus we found the point $\left(2, -\frac{16}{3}\right)$ on this line. Although the point we found is correct, its $y-$coordinate is not an integer. This makes the plotting of this point more difficult and also inaccurate. Whenever possible, we should rely on graphing lattice points. **A lattice point is a point of whose both coordinates are integers.**

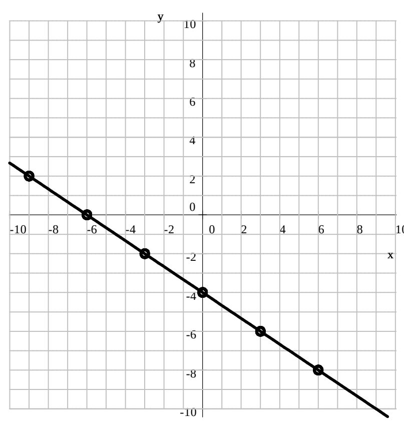
Let $x = 3$. We will compute the value of y.

$$\begin{aligned} y &= \ ?\ \text{if}\ \ x = 3 \\ x &= \ \ \text{and } 2x + 3y = -12 \quad \Longrightarrow \quad 2(3) + 3y = -12 \end{aligned}$$

$$\begin{aligned} 2(3)+3y &= -12 && \text{solve for } y \\ 3y+6 &= -12 && \text{subtract } 6 \\ 3y &= -18 && \text{divide by } 3 \\ y &= -6 \end{aligned}$$

If $x = 3$, then $y = -6$. Thus we found the point $(3, -6)$ on this line.
We continue to find additional points in this manner. Notice that we get lattice points if we use numbers for x that are divisible by 3. We organize the results in a table. We plot the lattice points and connect the dots.

x	y	$\Longrightarrow$	$P(x, y)$
-6	0		$(-6, 0)$
-3	-2		$(-3, -2)$
0	-4		$(0, -4)$
3	-6		$(3, -6)$
-9	2		$(-9, 2)$
-12	4		$(-12, 4)$



The point where the graph intersects the $x-$axis is called the $x-$intercept. The point where the graph intersects the $y-$axis is called the $y-$intercept. This line's $x-$intercept is $(-6, 0)$ and $y-$intercept is $(0, -4)$.

4. Graph the line $y = -2$

Solution: We freely select any value for x. We find the $y-$coordinate of the point using the equation of the line. We substitute the value for x and solve the equation for y.

$$\begin{aligned} y &= ? \text{ if } x = 0 \\ x &= 0 \text{ and } y = -2 \implies P(0, -2) \end{aligned}$$

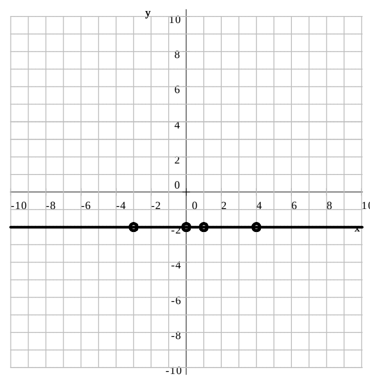
If $x = 0$, then $y = -2$. Thus we found the point $(0, -2)$ on this line. We repeat the process with other values for x to find other points on the line.

$$\begin{aligned} y &= ? \text{ if } x = -3 \\ x &= -3 \text{ and } y = -2 \implies P(-3, -2) \end{aligned}$$

If $x = -3$, then $y = -2$. Thus we found the point $(-3, -2)$ on this line.

We continue to find additional points in this manner. It is clear that no matter what the value of x is, y will always be -2. We organize the results in a table. We plot the lattice points and connect the dots.

x	y	$\implies$	$P(x, y)$
-3	-2		$(-3, -2)$
0	-2		$(0, -2)$
1	-2		$(1, -2)$
4	-2		$(4, -2)$



This line's $y-$intercept is $(0, -2)$ and it does not have an $x-$intercept.

Method 2 - Graphing using the slope-intercept form

The slope-intercept form of a line's equation enables us to plot a line quickly and with very little computation. Every non-vertical line has a slope-intercept form.

5. Graph the line $3x - 4y = 12$.

Solution:

Step 1. We bring the equation to its slope-intercept form, $y = mx + b$.

We will do that by solving for y in $3x - 4y = 12$.

$$\begin{aligned} 3x - 4y &= 12 && \text{add } 4y \\ 3x &= 4y + 12 && \text{subtract } 12 \\ 3x - 12 &= 4y && \text{divide by } 4 \\ \frac{3x-12}{4} &= y && \\ y &= \frac{3x-12}{4} = \frac{3x}{4} - \frac{12}{4} = \frac{3}{4}x - 3 && \end{aligned}$$

The slope-intercept form of the equation is $y = \frac{3}{4}x - 3$. This form of the equation shows that the slope of this line is $m = \frac{3}{4}$, the coefficient of x in the equation, and the $y-$intercept of it is $(0, -3)$ (just substitute $x = 0$ into the equation).

Step 2. We graph the $y-$intercept of the line.

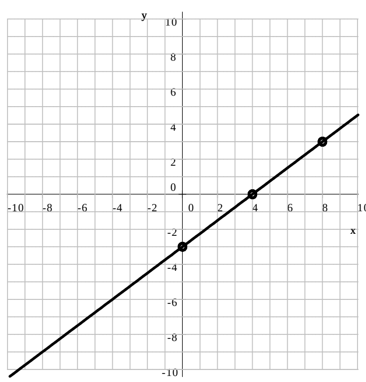
Consider the slope-intercept form of the line, $y = \frac{3}{4}x - 3$. We obtain the $y-$intercept by substituting $x = 0$ into this. Clearly, the $y-$intercept is $(0, -3)$. In general, the $y-$intercept of the line $y = mx + b$ is $(0, b)$.

Step 3. Graph additional points using the slope of the line.

The slope of this line is $m = \frac{3}{4}$, the coefficient of x in the slope-intercept form of the line. If the slope is a fraction, the denominator tells us how many units we step to the right, and the numerator tells us how many units we step up (or down if the slope is negative).

$$\frac{3}{4} \implies \frac{3 \text{ up}}{4 \text{ to the right}}$$

We start at the $y-$intercept, $(0, -3)$ and to plot additional points, we step 4 to the right, and 3 up. We repeat this process several times to plot enough points.



6. Graph the line $x + 2y = 4$

Solution:

Step 1. We bring the equation to its slope-intercept form, $y = mx + b$ by solving for y in $x + 2y = 4$.

$$\begin{array}{rcll} x + 2y &=& 4 & \text{subtract } x \\ 2y &=& -x + 4 & \text{divide by } 2 \\ y &=& \dfrac{-x+4}{2} = \dfrac{-x}{2} + \dfrac{4}{2} = -\dfrac{1}{2}x + 2 & \end{array}$$

The slope-intercept form of the equation is $y = -\dfrac{1}{2}x + 2$.
Step 2. We graph the $y-$intercept of the line.

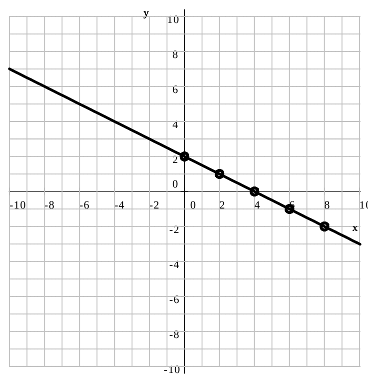
We obtain the $y-$intercept by substituting $x = 0$ into the slope-intercept form of the line, $y = -\dfrac{1}{2}x + 2$. Clearly, the $y-$intercept is $(0, 2)$.

Step 3. Graph additional points using the slope of the line.

The slope of this line is $m = -\dfrac{1}{2}$, the coefficient of x in the slope-intercept form of the line. If the slope is a fraction, the denominator tells us how many units we step to the right, and the numerator tells us how many units we step down (since the slope is negative).

$$-\frac{1}{2} = \frac{-1}{2} \quad \Longrightarrow \quad \frac{1 \text{ down}}{2 \text{ to the right}}$$

We start at the $y-$intercept, $(0, 2)$ and to plot additional points, we step 2 to the right, 1 down. We repeat this process several times to plot enough points.



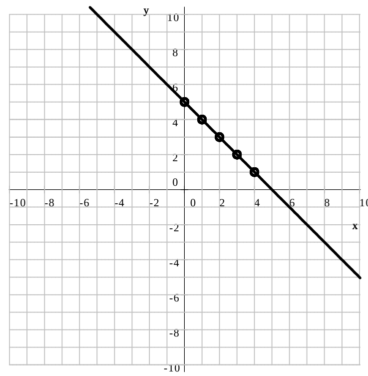
7. Graph the line $y = -x + 5$

Solution: We don't need to bring the equation to its slope-intercept form because it is given to us in that form. The slope of this line is $m = -1$, and the $y-$intercept of it is $(0, 5)$. We graph that point first. We graph additional points using the slope of the line.

The slope of this line is $m = -1$. If the slope is a not fraction, we can always divide it by 1 to create one.

$$-1 = \frac{-1}{1} \quad \Longrightarrow \quad \frac{1 \text{ down}}{1 \text{ to the right}}$$

We start at the $y-$intercept, $(0, 5)$ and to plot additional points, we step 1 to the right, 1 down. We repeat this process several times to plot enough points.



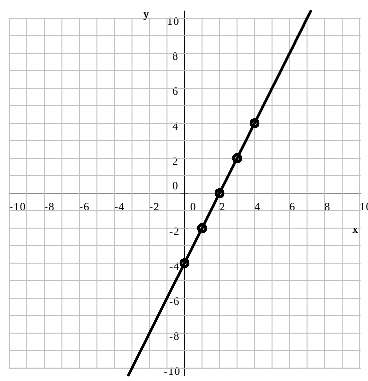
8. Graph the line $y = 2x - 4$

Solution: We don't need to bring the equation to its slope-intercept form because it is given to us in that form. The slope of this line is $m = 2$, and the $y-$intercept of it is $(0, -4)$. We graph that point first. We graph additional points using the slope of the line.

The slope of this line is $m = 2$. If the slope is a not fraction, we can always divide it by 1 to create one.

$$2 = \frac{2}{1} \implies \frac{2 \text{ up}}{1 \text{ to the right}}$$

We start at the $y-$intercept, $(0, -4)$ and to plot additional points, we step 1 to the right, 2 up. We repeat this process several times to plot enough points.



Practice Problems

1. Graph each of the following lines.

a) $3x + 2y = 6$

b) $x = -4$

c) $y = \dfrac{2}{5}x - 3$

d) $2x - 3y = 10$

e) $y = 1$

f) $y = 3x + 6$

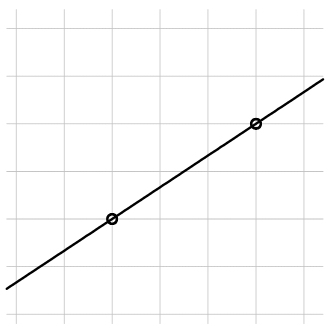
g) $3x + 5y = -30$

h) $2x - y = 7$

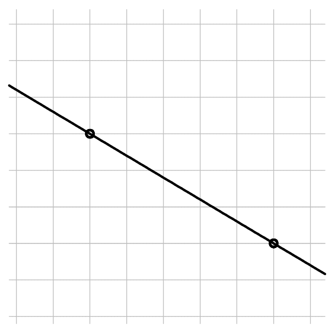
i) $y = \dfrac{1}{3}x$

2. Determine the slope of each of the following lines, based on their graphs.

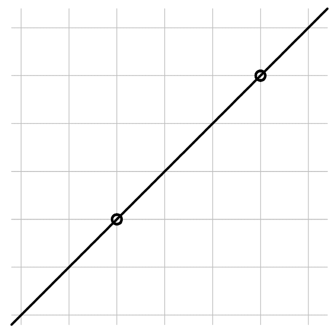
a)

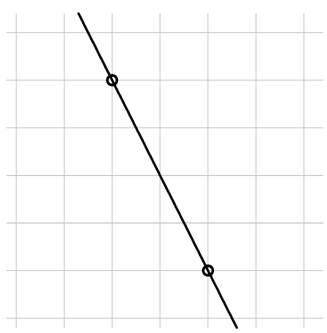


b)



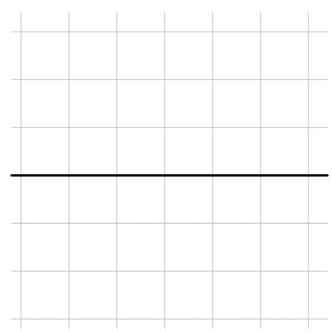
c)





d)

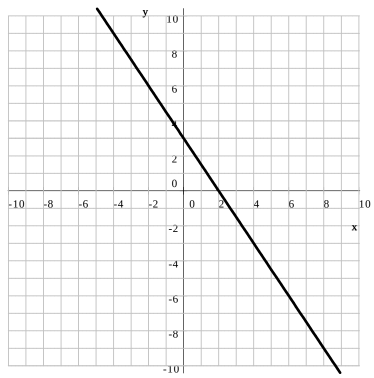
e)



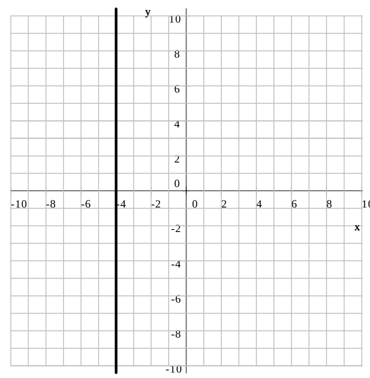
f)

Practice Problems - Answers

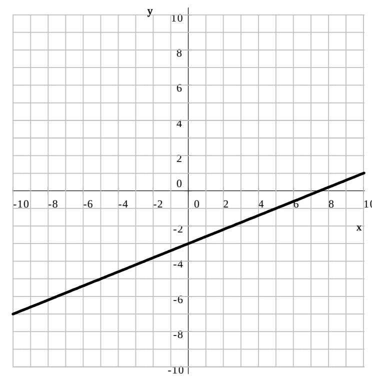
1. a) $3x + 2y = 6$



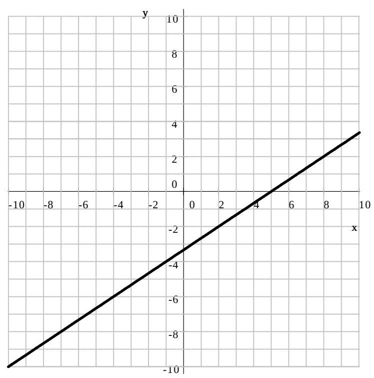
b) $x = -4$



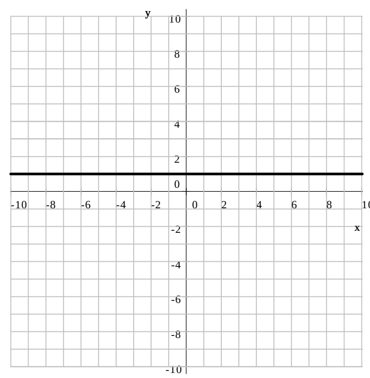
c) $y = \frac{2}{5}x - 3$



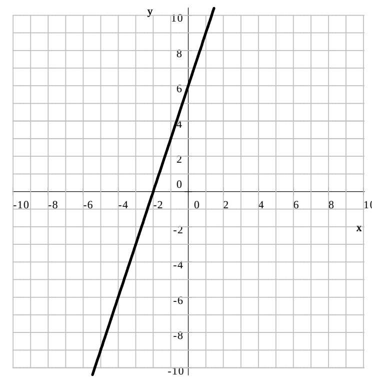
d) $2x - 3y = 10$



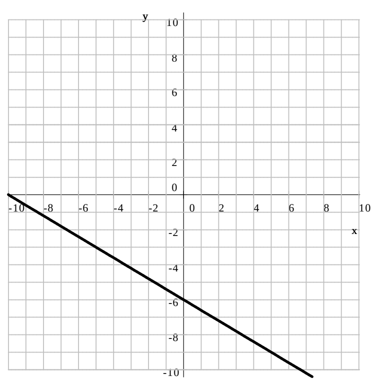
e) $y = 1$



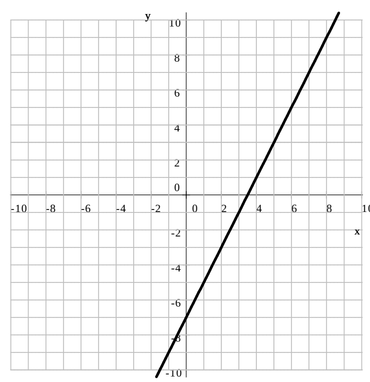
f) $y = 3x + 6$



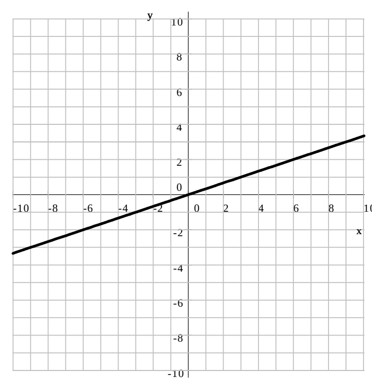
g) $3x + 5y = -30$



h) $2x - y = 7$



i) $y = \frac{1}{3}x$



2. a) $m = \frac{2}{3}$ b) $m = -\frac{3}{5}$ c) $m = 1$ d) $m = -2$ e) $m = \frac{1}{2}$ f) $m = 0$

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