

Part 1 - The unknown occurs only on one side

Solve each of the following equations. Make sure to check your solutions.

1. $2x - 5 = 17$ 11

Solution:

$$\begin{aligned} 2x - 5 &= 17 && \text{add 5 to both sides} \\ 2x &= 22 && \text{divide by 2} \\ x &= 11 \end{aligned}$$

We check: if $x = 11$, then

$$\begin{aligned} \text{RHS} &= 2(11) - 5 = 22 - 5 = 17 \\ \text{LHS} &= 17 \end{aligned}$$

Thus our solution, $x = 11$ is correct.

2. $\frac{3}{8}x + \left(1\frac{4}{5}\right) = \frac{3}{10}$ -4

Solution: this is a very simple equation, much like $2x + 1 = 7$, only the numbers are fractions. But the principles and operations regarding equations are the same.

$$\begin{aligned} \frac{3}{8}x + \left(1\frac{4}{5}\right) &= \frac{3}{10} && \text{convert mixed number to improper fraction} \\ \frac{3}{8}x + \frac{9}{5} &= \frac{3}{10} && \text{subtract } \frac{9}{5} \text{ from both sides; } \frac{3}{10} - \frac{9}{5} = \frac{3}{10} - \frac{18}{10} = \frac{3-18}{10} = \frac{-15}{10} = \frac{-3}{2} \\ \frac{3}{8}x &= \frac{-3}{2} && \text{divide both sides by } \frac{3}{8} \\ x &= -4 && \left(\frac{-3}{2}\right) \div \left(\frac{3}{8}\right) = \frac{-3}{2} \cdot \frac{8}{3} = \frac{-24}{6} = -4 \end{aligned}$$

We check:

$$\begin{aligned} \text{RHS} &= \frac{3}{8}(-4) + \left(1\frac{4}{5}\right) = \frac{3}{8} \cdot \frac{-4}{1} + \frac{9}{5} = \frac{-12}{8} + \frac{9}{5} = \frac{-3}{2} + \frac{9}{5} = \frac{-15}{10} + \frac{18}{10} = \frac{3}{10} \\ \text{LHS} &= \frac{3}{10} \end{aligned}$$

Thus our solution, -4 is correct.

3. $\frac{a-10}{5} = -3$ -3

Solution:

$$\begin{aligned} \frac{a-10}{5} &= -3 && \text{multiply both sides by 5} \\ a-10 &= -15 && \text{add 10 to both sides} \\ a &= -5 \end{aligned}$$

We check: if $a = -5$, then

$$\begin{aligned} \text{LHS} &= \frac{-5-10}{5} = \frac{-15}{5} = -3 \\ \text{RHS} &= -3 \end{aligned}$$

Thus our solution is correct.

4. $\frac{3t}{4} - 10 = -4$ 8
Solution:

$$\begin{aligned}\frac{3t}{4} - 10 &= -4 && \text{add } 10 \text{ to both sides} \\ \frac{3t}{4} &= 6 && \text{multiply both sides by } 4 \\ 3t &= 24 && \text{divide both sides by } 3 \\ t &= 8\end{aligned}$$

We check: if $t = 8$, then

$$\begin{aligned}\text{RHS} &= \frac{3t}{4} - 10 = \frac{3(8)}{4} - 10 = \frac{24}{4} - 10 = 6 - 10 = -4 \\ \text{LHS} &= -4\end{aligned}$$

Thus our solution, $t = 8$ is correct.

5. $\frac{t-5}{12} = 4$
Solution:

$$\begin{aligned}\frac{t-5}{12} &= 4 && \text{multiply both sides by } 12 \\ t-5 &= 48 && \text{add } 5 \text{ to both sides} \\ t &= 53\end{aligned}$$

We check: if $t = 53$, then

$$\text{RHS} = \frac{53-5}{12} = \frac{48}{12} = 4 = \text{LHS}$$

Thus our solution, $t = 53$ is correct.

6. $\frac{2x-7}{3} = -1$ 2
Solution: We apply all operations to both sides.

$$\begin{aligned}\frac{2x-7}{3} &= -1 && \text{multiply by } 3 \\ 2x-7 &= -3 && \text{add } 7 \\ 2x &= 4 && \text{divide by } 2 \\ x &= 2\end{aligned}$$

We check:

$$\text{LHS} = \frac{2(2)-7}{3} = \frac{4-7}{3} = \frac{-3}{3} = -1 = \text{RHS}$$

Thus our solution, $x = 2$ is correct.

7. $\frac{x+8}{3} = -2$ -14
Solution: We apply all operations to both sides.

$$\begin{aligned}\frac{x+8}{3} &= -2 && \text{multiply by } 3 \\ x+8 &= -6 && \text{subtract } 8 \\ x &= -14\end{aligned}$$

We check:

$$\text{LHS} = \frac{-14+8}{3} = \frac{-6}{3} = -2 = \text{RHS}$$

Thus our solution, $x = -14$ is correct.

8. $\frac{x}{3} + 8 = -2$ **- 30**

Solution: We apply all operations to both sides.

$$\begin{array}{rcl} \frac{x}{3} + 8 & = & -2 \quad \text{subtract } 8 \\ \frac{x}{3} & = & -10 \quad \text{multiply by } 3 \\ x & = & -30 \end{array}$$

We check:

$$\text{LHS} = \frac{-30}{3} + 8 = -10 + 8 = -2 = \text{RHS}$$

Thus our solution, $x = -30$ is correct.

9. $\frac{1}{5}x - \frac{2}{3} = \frac{26}{15}$ **12**

Solution:

$$\begin{array}{rcl} \frac{1}{5}x - \frac{2}{3} & = & \frac{26}{15} \quad \text{add } \frac{2}{3} \text{ to both sides} \\ \frac{1}{5}x & = & \frac{12}{5} \quad \text{divide by } \frac{1}{5} \\ x & = & 12 \end{array} \quad \begin{array}{l} \frac{26}{15} + \frac{2}{3} = \frac{26}{15} + \frac{2 \cdot 5}{3 \cdot 5} = \\ \frac{26}{15} + \frac{10}{15} = \frac{36}{15} = \frac{\cancel{3} \cdot 12}{\cancel{3} \cdot 5} = \frac{12}{5} \\ \frac{12}{\frac{1}{5}} = \frac{12}{1} \cdot \frac{5}{1} = \frac{12 \cdot \cancel{5}}{1 \cdot \cancel{5}} = \frac{12}{1} = 12 \end{array}$$

We check: if $x = 12$, then

$$\begin{array}{l} \text{LHS} = \frac{1}{5} \cdot 12 - \frac{2}{3} = \frac{1}{5} \cdot \frac{12}{1} - \frac{2}{3} = \frac{12}{5} - \frac{2}{3} = \frac{12 \cdot 3}{5 \cdot 3} - \frac{2 \cdot 5}{3 \cdot 5} = \frac{36}{15} - \frac{10}{15} = \frac{26}{15} \\ \text{RHS} = \frac{26}{15} \end{array}$$

Thus our solution, $x = 12$ is correct.

Practice Problems - Part 1

Solve each of the following equations. Make sure to check your solutions.

1. $2x - 3 = -11$

7. $\frac{1}{3}x + \frac{2}{5} = -\frac{34}{15}$

11. $\frac{2x - 1}{7} = -3$

2. $-2x - 3 = 7$

8. $5x - 6 = -6$

12. $\frac{x}{7} - 1 = -3$

4. $\frac{x - 3}{7} = -2$

9. $\frac{x + \frac{3}{8}}{\frac{4}{2 \cdot 5}} = \frac{5}{16}$

13. $\frac{2}{3}b + \frac{3}{5} = -\frac{1}{15}$

5. $\frac{x}{7} - 3 = -1$

10. $-x + 5 = -7$

14. $\frac{a + 1}{4} = -9$

Part 2 - The unknown occurs on both sides

1. $2x + 3 = 4x + 9$ -3

Solution:

$$\begin{array}{rcll}
 2x + 3 & = & 4x + 9 & \text{subtract } 2x \text{ from both sides} \\
 3 & = & 2x + 9 & \text{subtract } 9 \text{ from both sides} \\
 -6 & = & 2x & \text{divide both sides by 2} \\
 -3 & = & x &
 \end{array}$$

We check: if $x = -3$, then

$$\begin{array}{lcl}
 \text{RHS} & = & 2(-3) + 3 = -6 + 3 = -3 \\
 \text{LHS} & = & 4(-3) + 9 = -12 + 9 = -3
 \end{array}$$

Thus our solution, $x = -3$ is correct.

2. Solve: $3w - 5 = 5(w + 1)$ -5

Solution: we first apply the law of distributivity to simplify the right-hand side.

$$\begin{array}{rcll}
 3w - 5 & = & 5(w + 1) & \\
 3w - 5 & = & 5w + 5 & \text{subtract } 3w \text{ from both sides} \\
 -5 & = & 2w + 5 & \text{subtract } 5 \text{ from both sides} \\
 -10 & = & 2w & \text{divide both sides by 2} \\
 -5 & = & w &
 \end{array}$$

We check. If $w = -5$, then

$$\begin{array}{lcl}
 \text{LHS} & = & 3(-5) - 5 = -15 - 5 = -20 \\
 \text{RHS} & = & 5((-5) + 1) = 5(-4) = -20
 \end{array}$$

Thus our solution, $w = -5$ is correct.

3. $3y - 9 = -2y + 4$ $\frac{13}{5}$

Solution:

$$\begin{array}{rcll}
 3y - 9 & = & -2y + 4 & \text{add } 2y \text{ to both sides} \\
 5y - 9 & = & 4 & \text{add } 9 \text{ to both sides} \\
 5y & = & 13 & \text{divide both sides by 5} \\
 y & = & \frac{13}{5} &
 \end{array}$$

We check. If $x = \frac{13}{5}$, then

$$\begin{array}{lcl}
 \text{LHS} & = & 3\left(\frac{13}{5}\right) - 9 = \frac{3}{1} \cdot \frac{13}{5} - 9 = \frac{39}{5} - \frac{9}{1} = \frac{39}{5} - \frac{45}{5} = \frac{-6}{5} = -\frac{6}{5} \\
 \text{RHS} & = & -2\left(\frac{13}{5}\right) + 4 = \frac{-2}{1} \cdot \frac{13}{5} + \frac{4}{1} = \frac{-26}{5} + \frac{20}{5} = \frac{-6}{5} = -\frac{6}{5}
 \end{array}$$

Thus $x = \frac{13}{5}$ is the correct solution.

4. $4 - x = 3(x - 7)$ $\frac{25}{4}$

Solution: We first apply the distributive law to simplify the right-hand side.

$$\begin{aligned}
 4 - x &= 3(x - 7) && \text{distribute } 3 \\
 4 - x &= 3x - 21 && \text{add } x \text{ to both sides} \\
 4 &= 4x - 21 && \text{add } 21 \text{ to both sides} \\
 25 &= 4x && \text{divide both sides by } 4 \\
 \frac{25}{4} &= x
 \end{aligned}$$

We check. If $x = \frac{25}{4}$, then

$$\begin{aligned}
 \text{LHS} &= 4 - x = 4 - \frac{25}{4} = \frac{4}{1} - \frac{25}{4} = \frac{16}{4} - \frac{25}{4} = \frac{16 - 25}{4} = \frac{-9}{4} = -\frac{9}{4} \\
 \text{RHS} &= 3(x - 7) = 3\left(\frac{25}{4} - 7\right) = 3\left(\frac{25}{4} - \frac{7}{1}\right) = 3\left(\frac{25}{4} - \frac{28}{4}\right) = 3\left(\frac{25 - 28}{4}\right) \\
 &= 3\left(\frac{-3}{4}\right) = \frac{3}{1} \cdot \frac{-3}{4} = \frac{-9}{4} = -\frac{9}{4}
 \end{aligned}$$

Thus our solution, $x = \frac{25}{4}$ is correct.

5. $7(j - 5) + 9 = 2(-2j + 5) + 5j$ 6

Solution:

$$\begin{aligned}
 7(j - 5) + 9 &= 2(-2j + 5) + 5j && \text{distribute on both sides} \\
 7j - 35 + 9 &= -4j + 10 + 5j && \text{combine like terms} \\
 7j - 26 &= j + 10 && \text{subtract } j \\
 6j - 26 &= 10 && \text{add } 26 \\
 6j &= 36 && \text{divide by } 6 \\
 j &= 6
 \end{aligned}$$

We check: if $j = 6$, then

$$\begin{aligned}
 \text{LHS} &= 7(6 - 5) + 9 = 7 \cdot 1 + 9 = 7 + 9 = 16 \\
 \text{RHS} &= 2(-2 \cdot 6 + 5) + 5 \cdot 6 = 2(-12 + 5) + 30 = 2(-7) + 30 = -14 + 30 = 16
 \end{aligned}$$

Thus our solution is correct.

6. $3(x - 5) - 5(x - 1) = -2x + 1$ **no solution**

Solution:

$$\begin{aligned}
 3(x - 5) - 5(x - 1) &= -2x + 1 && \text{multiply out parentheses} \\
 3x - 15 - 5x + 45 &= -2x + 1 && \text{combine like terms} \\
 -2x + 30 &= -2x + 1 && \text{add } 2x \\
 30 &= 1
 \end{aligned}$$

Since x disappeared from the equation and we are left with an unconditionally false statement, there is no solution for this equation. This type of an equation is called a contradiction.

7. $\frac{3-x}{4} - \frac{10-3x}{5} = x+2$ -5
 Solution:

$$\begin{aligned}
 \frac{3-x}{4} - \frac{10-3x}{5} &= x+2 && \text{make everything a fraction} \\
 \frac{3-x}{4} - \frac{10-3x}{5} &= \frac{x+2}{1} && \text{common denominator} \\
 \frac{5(3-x)}{20} - \frac{4(10-3x)}{20} &= \frac{20(x+2)}{20} && \text{multiply by 20} \\
 5(3-x) - 4(10-3x) &= 20(x+2) && \text{distribute} \\
 15 - 5x - 40 + 12x &= 20x + 40 && \text{combine like terms} \\
 7x - 25 &= 20x + 40 && \text{subtract } 7x \\
 -25 &= 13x + 40 && \text{subtract } 40 \\
 -65 &= 13x && \text{divide by 13} \\
 -5 &= x
 \end{aligned}$$

We check:

$$\begin{aligned}
 \text{LHS} &= \frac{3 - (-5)}{4} - \frac{10 - 3(-5)}{5} = \frac{8}{4} - \frac{25}{5} = 2 - 5 = -3 \\
 \text{RHS} &= -5 + 2 = -3
 \end{aligned}$$

Thus our solution, -5 is correct.

8. $\frac{3x+17}{2} = x-1 + \frac{x+19}{2}$ identity, all numbers are solution
 Solution:

$$\begin{aligned}
 \frac{3x+17}{2} &= x-1 + \frac{x+19}{2} && \text{express everything as a fraction} \\
 \frac{3x+17}{2} &= \frac{x-1}{1} + \frac{x+19}{2} && \text{bring everything to the common denominator} \\
 \frac{3x+17}{2} &= \frac{2(x-1)}{2} + \frac{x+19}{2} && \text{add fractions on right hand side} \\
 \frac{3x+17}{2} &= \frac{2(x-1) + x+19}{2} && \text{multiply out parentheses} \\
 \frac{3x+17}{2} &= \frac{2x-2+x+19}{2} && \text{combine like terms} \\
 \frac{3x+17}{2} &= \frac{3x+17}{2} && \text{multiply by 2} \\
 3x+17 &= 3x+17
 \end{aligned}$$

Because the left hand side is now identical to the right hand side, this equation is an identity, and all real numbers are solution.

9. $\frac{2}{3}(x-7) = \frac{4}{5}(x+1)$ **-41**
 Solution:

$$\begin{aligned}\frac{2}{3}(x-7) &= \frac{4}{5}(x+1) \\ \frac{2}{3} \cdot \frac{x-7}{1} &= \frac{4}{5} \cdot \frac{x+1}{1} \\ \frac{2(x-7)}{3} &= \frac{4(x+1)}{5} && \text{bring fractions to common denominator} \\ \frac{5 \cdot 2(x-7)}{15} &= \frac{3 \cdot 4(x+1)}{15} && \text{multiply both sides by 15}\end{aligned}$$

$$\begin{aligned}10(x-7) &= 12(x+1) && \text{multiply out parentheses} \\ 10x - 70 &= 12x + 12 && \text{subtract } 10x \\ -70 &= 2x + 12 && \text{subtract 12} \\ -82 &= 2x && \text{divide by 2} \\ -41 &= x\end{aligned}$$

We check:

$$\begin{aligned}\text{LHS} &= \frac{2}{3}(-41-7) = \frac{2}{3}(-48) = -32 \\ \text{RHS} &= \frac{4}{5}(-41+1) = \frac{4}{5}(-40) = -32\end{aligned}$$

Thus our solution, -41 is correct.

10. $\frac{x+2}{4} - \frac{x-3}{5} = 20 - x$ **18**
 Solution:

$$\begin{aligned}\frac{x+2}{4} - \frac{x-3}{5} &= 20 - x && \text{make everything a fraction} \\ \frac{x+2}{4} - \frac{x-3}{5} &= \frac{20-x}{1} && \text{common denominator is 20} \\ \frac{5(x+2)}{20} - \frac{4(x-3)}{20} &= \frac{20(20-x)}{20} && \text{multiply by 20} \\ 5(x+2) - 4(x-3) &= 20(20-x) && \text{distribute} \\ 5x + 10 - 4x + 12 &= 400 - 20x && \text{combine like terms} \\ x + 22 &= -20x + 400 && \text{add } 20x \\ 21x + 22 &= 400 && \text{subtract 22} \\ 21x &= 378 && \text{divide by 21} \\ x &= 18\end{aligned}$$

We check. If $x = 18$, then

$$\begin{aligned}\text{LHS} &= \frac{18+2}{4} - \frac{18-3}{5} = \frac{20}{4} - \frac{15}{5} = 5 - 3 = 2 \\ \text{RHS} &= 20 - 18 = 2\end{aligned}$$

Thus 18 is indeed the solution.

Practice Problems - Part 2

1. $5x - 3 = x + 9$
2. $-x + 13 = 2x + 1$
3. $-2x + 4 = 5x - 10$
4. $5x - 7 = 6x + 8$
5. $8x - 1 = 3x + 19$
6. $-7x - 1 = 3x - 21$
7. $3(x - 4) + 5(x + 8) = 2(x - 1)$
8. $3(x - 4) = 2(x + 5)$
9. $4(5x + 1) = 6x + 4$
10. $3(2x - 7) - 2(5x + 2) = -5x - 30$
11. $a - 3 = 5(a - 1) - 2$
12. $3y - 2 = -2y + 18$
13. $2(b + 1) - 5(b - 3) = 2(b - 7) + 1$
14. $3(2x - 1) - 5(2 - x) = 4(x - 1) + 5$
15. $5(x - 1) - 3(x + 1) = 3x - 8$
16. $5(x - 1) - 3(-x + 1) = -3 + 8x$
17. $-2x - (3x - 1) = 2(5 - 3x)$
18. $3(x - 4) - 4(x - 3) = 3(x - 2) + 2(3 - x)$
19. $\frac{3x - 1}{5} - \frac{7 - x}{3} = 2x + 6$
20. $8(x - 3) - 3(5 - 2x) = x$
21. $\frac{3x - 1}{4} + \frac{8 - 4x}{3} = -3 - x$
22. $\frac{3x - 2}{5} + \frac{x + 4}{3} = \frac{14(x + 1)}{15}$
23. $\frac{3}{8}x + 1\frac{4}{5} = \frac{1}{4}x + 1\frac{3}{10}$
24. $\frac{2x + 1}{3} - \frac{3 - x}{2} = x - 2$
25. $\frac{2}{3}x - 1 = -\frac{2}{3}\left(x + \frac{1}{2}\right)$

Answers for Practice Problems - Part 1

- 1.) -4 2.) -5 3.) 4 4.) -11 5.) 14 6.) -4 7.) -8 8.) 0 9.) $\frac{1}{2}$ 10.) 12
11.) -10 12.) -14 13.) -1 14.) -37

Answers for Practice Problems - Part 2

- 1.) 3 2.) 4 3.) 2 4.) -15 5.) 4 6.) 2 7.) -15 8.) 22 9.) 0 10.) -5
11.) 1 12.) 4 13.) 6 14.) 2 15.) 0 16.) contradiction, there is no solution 17.) 9
18.) 0 19.) -8 20.) 3 21.) -13 22.) identity, all numbers are solution 23.) -4
24.) -5 25.) $\frac{1}{2}$

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