

Chapter 1 : Solving Linear Equations

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Solving Linear Equations - One Step Equations

Objective: Solve one step linear equations by balancing using inverse operations

Solving linear equations is an important and fundamental skill in algebra. In algebra, we are often presented with a problem where the answer is known, but part of the problem is missing. The missing part of the problem is what we seek to find. An example of such a problem is shown below.

Example 41.

$$4x + 16 = -4$$

Notice the above problem has a missing part, or unknown, that is marked by x . If we are given that the solution to this equation is -5 , it could be plugged into the equation, replacing the x with -5 . This is shown in Example 2.

Example 42.

$$\begin{array}{ll} 4(-5) + 16 = -4 & \text{Multiply } 4(-5) \\ -20 + 16 = -4 & \text{Add } -20 + 16 \\ -4 = -4 & \text{True!} \end{array}$$

Now the equation comes out to a true statement! Notice also that if another number, for example, 3, was plugged in, we would not get a true statement as seen in Example 3.

Example 43.

$$\begin{array}{ll} 4(3) + 16 = -4 & \text{Multiply } 4(3) \\ 12 + 16 = -4 & \text{Add } 12 + 16 \\ 28 \neq -4 & \text{False!} \end{array}$$

Due to the fact that this is not a true statement, this demonstrates that 3 is not the solution. However, depending on the complexity of the problem, this “guess and check” method is not very efficient. Thus, we take a more algebraic approach to solving equations. Here we will focus on what are called “one-step equations” or equations that only require one step to solve. While these equations often seem very fundamental, it is important to master the pattern for solving these problems so we can solve more complex problems.

Addition Problems

To solve equations, the general rule is to do the opposite. For example, consider the following example.

Example 44.

$$\begin{array}{ll} x + 7 = -5 & \text{The 7 is added to the } x \\ \underline{-7 \quad -7} & \text{Subtract 7 from both sides to get rid of it} \\ x = -12 & \text{Our solution!} \end{array}$$

Then we get our solution, $x = -12$. The same process is used in each of the following examples.

Example 45.

$$\begin{array}{lll} 4 + x = 8 & 7 = x + 9 & 5 = 8 + x \\ \underline{-4 \quad -4} & \underline{-9 \quad -9} & \underline{-8 \quad -8} \\ x = 4 & -2 = x & -3 = x \end{array}$$

Table 1. Addition Examples

Subtraction Problems

In a subtraction problem, we get rid of negative numbers by adding them to both sides of the equation. For example, consider the following example.

Example 46.

$$\begin{array}{ll} x - 5 = 4 & \text{The 5 is negative, or subtracted from } x \\ \underline{+5 \quad +5} & \text{Add 5 to both sides} \\ x = 9 & \text{Our Solution!} \end{array}$$

Then we get our solution $x = 9$. The same process is used in each of the following examples. Notice that each time we are getting rid of a negative number by adding.

Example 47.

$$\begin{array}{r} -6 + x = -2 \\ + 6 \quad + 6 \\ \hline x = 4 \end{array}$$

$$\begin{array}{r} -10 = x - 7 \\ + 7 \quad + 7 \\ \hline -3 = x \end{array}$$

$$\begin{array}{r} 5 = -8 + x \\ + 8 \quad + 8 \\ \hline 13 = x \end{array}$$

Table 2. Subtraction Examples

Multiplication Problems

With a multiplication problem, we get rid of the number by dividing on both sides. For example consider the following example.

Example 48.

$$\begin{array}{r} 4x = 20 \\ \hline 4 \quad 4 \\ \hline x = 5 \end{array} \quad \begin{array}{l} \text{Variable is multiplied by 4} \\ \text{Divide both sides by 4} \\ \text{Our solution!} \end{array}$$

Then we get our solution $x = 5$

With multiplication problems it is very important that care is taken with signs. If x is multiplied by a negative then we will divide by a negative. This is shown in example 9.

Example 49.

$$\begin{array}{r} -5x = 30 \\ \hline -5 \quad -5 \\ \hline x = -6 \end{array} \quad \begin{array}{l} \text{Variable is multiplied by } -5 \\ \text{Divide both sides by } -5 \\ \text{Our Solution!} \end{array}$$

The same process is used in each of the following examples. Notice how negative and positive numbers are handled as each problem is solved.

Example 50.

$$\begin{array}{r} 8x = -24 \\ \frac{8}{8} \quad \frac{8}{8} \\ x = -3 \end{array}$$

$$\begin{array}{r} -4x = -20 \\ \frac{-4}{-4} \quad \frac{-20}{-4} \\ x = 5 \end{array}$$

$$\begin{array}{r} 42 = 7x \\ \frac{42}{7} = \frac{7x}{7} \\ 6 = x \end{array}$$

Table 3. Multiplication Examples

Division Problems:

In division problems, we get rid of the denominator by multiplying on both sides. For example consider our next example.

Example 51.

$$\begin{array}{ll} \frac{x}{5} = -3 & \text{Variable is divided by 5} \\ (5)\frac{x}{5} = -3(5) & \text{Multiply both sides by 5} \\ x = -15 & \text{Our Solution!} \end{array}$$

Then we get our solution $x = -15$. The same process is used in each of the following examples.

Example 52.

$$\begin{array}{lll} \frac{x}{-7} = -2 & \frac{x}{8} = 5 & \frac{x}{-4} = 9 \\ (-7)\frac{x}{-7} = -2(-7) & (8)\frac{x}{8} = 5(8) & (-4)\frac{x}{-4} = 9(-4) \\ x = 14 & x = 40 & x = -36 \end{array}$$

Table 4. Division Examples

The process described above is fundamental to solving equations. once this process is mastered, the problems we will see have several more steps. These problems may seem more complex, but the process and patterns used will remain the same.

World View Note: The study of algebra originally was called the “Cossic Art” from the Latin, the study of “things” (which we now call variables).

1.1 Practice - One Step Equations

Solve each equation.

1) $v + 9 = 16$

2) $14 = b + 3$

3) $x - 11 = -16$

4) $-14 = x - 18$

5) $30 = a + 20$

6) $-1 + k = 5$

7) $x - 7 = -26$

8) $-13 + p = -19$

9) $13 = n - 5$

10) $22 = 16 + m$

11) $340 = -17x$

12) $4r = -28$

13) $-9 = \frac{n}{12}$

14) $\frac{5}{9} = \frac{b}{9}$

15) $20v = -160$

16) $-20x = -80$

17) $340 = 20n$

18) $\frac{1}{2} = \frac{a}{8}$

19) $16x = 320$

20) $\frac{k}{13} = -16$

21) $-16 + n = -13$

22) $21 = x + 5$

23) $p - 8 = -21$

24) $m - 4 = -13$

25) $180 = 12x$

26) $3n = 24$

27) $20b = -200$

28) $-17 = \frac{x}{12}$

29) $\frac{r}{14} = \frac{5}{14}$

30) $n + 8 = 10$

31) $-7 = a + 4$

32) $v - 16 = -30$

33) $10 = x - 4$

34) $-15 = x - 16$

35) $13a = -143$

36) $-8k = 120$

37) $\frac{p}{20} = -12$

38) $-15 = \frac{x}{9}$

39) $9 + m = -7$

40) $-19 = \frac{n}{20}$

Linear Equations - Two-Step Equations

Objective: Solve two-step equations by balancing and using inverse operations.

After mastering the technique for solving equations that are simple one-step equations, we are ready to consider two-step equations. As we solve two-step equations, the important thing to remember is that everything works backwards! When working with one-step equations, we learned that in order to clear a “plus five” in the equation, we would subtract five from both sides. We learned that to clear “divided by seven” we multiply by seven on both sides. The same pattern applies to the order of operations. When solving for our variable x , we use order of operations backwards as well. This means we will add or subtract first, then multiply or divide second (then exponents, and finally any parentheses or grouping symbols, but that’s another lesson). So to solve the equation in the first example,

Example 53.

$$4x - 20 = -8$$

We have two numbers on the same side as the x . We need to move the 4 and the 20 to the other side. We know to move the four we need to divide, and to move the twenty we will add twenty to both sides. If order of operations is done backwards, we will add or subtract first. Therefore we will add 20 to both sides first. Once we are done with that, we will divide both sides by 4. The steps are shown below.

$4x - 20 = -8$	Start by focusing on the subtract 20
$\quad \underline{+ 20 + 20}$	Add 20 to both sides
$4x \quad = 12$	Now we focus on the 4 multiplied by x
$\underline{4} \quad \quad \underline{4}$	Divide both sides by 4
$x = 3$	Our Solution!

Notice in our next example when we replace the x with 3 we get a true statement.

$4(3) - 20 = -8$	Multiply $4(3)$
$12 - 20 = -8$	Subtract $12 - 20$
$-8 = -8$	True!

The same process is used to solve any two-step equations. Add or subtract first, then multiply or divide. Consider our next example and notice how the same process is applied.

Example 54.

$$\begin{array}{rcl}
 5x + 7 = & 7 & \text{Start by focusing on the plus 7} \\
 \underline{- 7} \quad \underline{- 7} & & \text{Subtract 7 from both sides} \\
 5x & = 0 & \text{Now focus on the multiplication by 5} \\
 \underline{5} \quad \underline{5} & & \text{Divide both sides by 5} \\
 x = & 0 & \text{Our Solution!}
 \end{array}$$

Notice the seven subtracted out completely! Many students get stuck on this point, do not forget that we have a number for “nothing left” and that number is zero. With this in mind the process is almost identical to our first example.

A common error students make with two-step equations is with negative signs. Remember the sign always stays with the number. Consider the following example.

Example 55.

$$\begin{array}{rcl}
 4 - 2x = 10 & & \text{Start by focusing on the positive 4} \\
 \underline{- 4} \quad \underline{- 4} & & \text{Subtract 4 from both sides} \\
 - 2x & = 6 & \text{Negative (subtraction) stays on the } 2x \\
 \underline{- 2} \quad \underline{- 2} & & \text{Divide by } - 2 \\
 x = & - 3 & \text{Our Solution!}
 \end{array}$$

The same is true even if there is no coefficient in front of the variable. Consider the next example.

Example 56.

$$\begin{array}{rcl}
 8 - x = 2 & & \text{Start by focusing on the positive 8} \\
 \underline{- 8} \quad \underline{- 8} & & \text{Subtract 8 from both sides} \\
 - x & = - 6 & \text{Negative (subtraction) stays on the } x \\
 - 1x & = - 6 & \text{Remember, no number in front of variable means 1}
 \end{array}$$

$$\begin{array}{r} \overline{-1} \quad \overline{-1} \\ x = 6 \end{array} \quad \begin{array}{l} \text{Divide both sides by } -1 \\ \text{Our Solution!} \end{array}$$

Solving two-step equations is a very important skill to master, as we study algebra. The first step is to add or subtract, the second is to multiply or divide. This pattern is seen in each of the following examples.

Example 57.

$$\begin{array}{r} -3x + 7 = -8 \\ \underline{-7 \quad -7} \\ -3x = -15 \\ \underline{-3 \quad -3} \\ x = 5 \end{array}$$

$$\begin{array}{r} -2 + 9x = 7 \\ \underline{+2 \quad +2} \\ 9x = 9 \\ \underline{9 \quad 9} \\ x = 1 \end{array}$$

$$\begin{array}{r} 8 = 2x + 10 \\ \underline{-10 \quad -10} \\ -2 = 2x \\ \underline{2 \quad 2} \\ -1 = x \end{array}$$

$$\begin{array}{r} 7 - 5x = 17 \\ \underline{-7 \quad -7} \\ -5x = 10 \\ \underline{-5 \quad -5} \\ x = -2 \end{array}$$

$$\begin{array}{r} -5 - 3x = -5 \\ \underline{+5 \quad +5} \\ -3x = 0 \\ \underline{-3 \quad -3} \\ x = 0 \end{array}$$

$$\begin{array}{r} -3 = \frac{x}{5} - 4 \\ \underline{+4 \quad +4} \\ (5)(1) = \frac{x}{5}(5) \\ 5 = x \end{array}$$

Table 5. Two-Step Equation Examples

As problems in algebra become more complex the process covered here will remain the same. In fact, as we solve problems like those in the next example, each one of them will have several steps to solve, but the last two steps are a two-step equation like we are solving here. This is why it is very important to master two-step equations now!

Example 58.

$$3x^2 + 4 - x + 6 \qquad \frac{1}{x-8} + \frac{1}{x} = \frac{1}{3} \qquad \sqrt{5x-5} + 1 = x \qquad \log_5(2x-4) = 1$$

World View Note: Persian mathematician Omar Khayyam would solve algebraic problems geometrically by intersecting graphs rather than solving them algebraically.

1.2 Practice - Two-Step Problems

Solve each equation.

1) $5 + \frac{n}{4} = 4$

3) $102 = -7r + 4$

5) $-8n + 3 = -77$

7) $0 = -6v$

9) $-8 = \frac{x}{5} - 6$

11) $0 = -7 + \frac{k}{2}$

13) $-12 + 3x = 0$

15) $24 = 2n - 8$

17) $2 = -12 + 2r$

19) $\frac{b}{3} + 7 = 10$

21) $152 = 8n + 64$

23) $-16 = 8a + 64$

25) $56 + 8k = 64$

27) $-2x + 4 = 22$

29) $-20 = 4p + 4$

31) $-5 = 3 + \frac{n}{2}$

33) $\frac{r}{8} - 6 = -5$

35) $-40 = 4n - 32$

37) $87 = 3 - 7v$

39) $-x + 1 = -11$

2) $-2 = -2m + 12$

4) $27 = 21 - 3x$

6) $-4 - b = 8$

8) $-2 + \frac{x}{2} = 4$

10) $-5 = \frac{a}{4} - 1$

12) $-6 = 15 + 3p$

14) $-5m + 2 = 27$

16) $-37 = 8 + 3x$

18) $-8 + \frac{n}{12} = -7$

20) $\frac{x}{1} - 8 = -8$

22) $-11 = -8 + \frac{v}{2}$

24) $-2x - 3 = -29$

26) $-4 - 3n = -16$

28) $67 = 5m - 8$

30) $9 = 8 + \frac{x}{6}$

32) $\frac{m}{4} - 1 = -2$

34) $-80 = 4x - 28$

36) $33 = 3b + 3$

38) $3x - 3 = -3$

40) $4 + \frac{a}{3} = 1$

Solving Linear Equations - General Equations

Objective: Solve general linear equations with variables on both sides.

Often as we are solving linear equations we will need to do some work to set them up into a form we are familiar with solving. This section will focus on manipulating an equation we are asked to solve in such a way that we can use our pattern for solving two-step equations to ultimately arrive at the solution.

One such issue that needs to be addressed is parenthesis. Often the parenthesis can get in the way of solving an otherwise easy problem. As you might expect we can get rid of the unwanted parenthesis by using the distributive property. This is shown in the following example. Notice the first step is distributing, then it is solved like any other two-step equation.

Example 59.

$$\begin{array}{ll}
 4(2x - 6) = 16 & \text{Distribute 4 through parenthesis} \\
 8x - 24 = 16 & \text{Focus on the subtraction first} \\
 \underline{+ 24 + 24} & \text{Add 24 to both sides} \\
 8x = 40 & \text{Now focus on the multiply by 8} \\
 \underline{8} \quad \underline{8} & \text{Divide both sides by 8} \\
 x = 5 & \text{Our Solution!}
 \end{array}$$

Often after we distribute there will be some like terms on one side of the equation. Example 2 shows distributing to clear the parenthesis and then combining like terms next. Notice we only combine like terms on the same side of the equation. Once we have done this, our next example solves just like any other two-step equation.

Example 60.

$$\begin{array}{ll}
 3(2x - 4) + 9 = 15 & \text{Distribute the 3 through the parenthesis} \\
 6x - 12 + 9 = 15 & \text{Combine like terms, } -12 + 9 \\
 6x - 3 = 15 & \text{Focus on the subtraction first} \\
 \underline{+ 3 + 3} & \text{Add 3 to both sides} \\
 6x = 18 & \text{Now focus on multiply by 6}
 \end{array}$$

$$\begin{array}{r} \overline{6} \quad \overline{6} \\ x = 3 \end{array} \quad \begin{array}{l} \text{Divide both sides by 6} \\ \text{Our Solution} \end{array}$$

A second type of problem that becomes a two-step equation after a bit of work is one where we see the variable on both sides. This is shown in the following example.

Example 61.

$$4x - 6 = 2x + 10$$

Notice here the x is on both the left and right sides of the equation. This can make it difficult to decide which side to work with. We fix this by moving one of the terms with x to the other side, much like we moved a constant term. It doesn't matter which term gets moved, $4x$ or $2x$, however, it would be the author's suggestion to move the smaller term (to avoid negative coefficients). For this reason we begin this problem by clearing the positive $2x$ by subtracting $2x$ from both sides.

$$\begin{array}{r} 4x - 6 = 2x + 10 \\ \underline{- 2x} \quad \underline{- 2x} \\ 2x - 6 = 10 \\ \underline{+ 6} \quad \underline{+ 6} \\ 2x = 16 \\ \underline{2} \quad \underline{2} \\ x = 8 \end{array} \quad \begin{array}{l} \text{Notice the variable on both sides} \\ \text{Subtract } 2x \text{ from both sides} \\ \text{Focus on the subtraction first} \\ \text{Add 6 to both sides} \\ \text{Focus on the multiplication by 2} \\ \text{Divide both sides by 2} \\ \text{Our Solution!} \end{array}$$

The previous example shows the check on this solution. Here the solution is plugged into the x on both the left and right sides before simplifying.

Example 62.

$$\begin{array}{r} 4(8) - 6 = 2(8) + 10 \\ 32 - 6 = 16 + 10 \\ 26 = 26 \end{array} \quad \begin{array}{l} \text{Multiply } 4(8) \text{ and } 2(8) \text{ first} \\ \text{Add and Subtract} \\ \text{True!} \end{array}$$

The next example illustrates the same process with negative coefficients. Notice first the smaller term with the variable is moved to the other side, this time by adding because the coefficient is negative.

Example 63.

$-3x + 9 = 6x - 27$	Notice the variable on both sides, $-3x$ is smaller
$\underline{+ 3x \quad + 3x}$	Add $3x$ to both sides
$9 = 9x - 27$	Focus on the subtraction by 27
$\underline{+ 27 \quad + 27}$	Add 27 to both sides
$36 = 9x$	Focus on the multiplication by 9
$\underline{9 \quad 9}$	Divide both sides by 9
$4 = x$	Our Solution

Linear equations can become particularly interesting when the two processes are combined. In the following problems we have parenthesis and the variable on both sides. Notice in each of the following examples we distribute, then combine like terms, then move the variable to one side of the equation.

Example 64.

$2(x - 5) + 3x = x + 18$	Distribute the 2 through parenthesis
$2x - 10 + 3x = x + 18$	Combine like terms $2x + 3x$
$5x - 10 = x + 18$	Notice the variable is on both sides
$\underline{- x \quad - x}$	Subtract x from both sides
$4x - 10 = 18$	Focus on the subtraction of 10
$\underline{+ 10 \quad + 10}$	Add 10 to both sides
$4x = 28$	Focus on multiplication by 4
$\underline{4 \quad 4}$	Divide both sides by 4
$x = 7$	Our Solution

Sometimes we may have to distribute more than once to clear several parenthesis. Remember to combine like terms after you distribute!

Example 65.

$3(4x - 5) - 4(2x + 1) = 5$	Distribute 3 and -4 through parenthesis
$12x - 15 - 8x - 4 = 5$	Combine like terms $12x - 8x$ and $-15 - 4$
$4x - 19 = 5$	Focus on subtraction of 19
$\underline{+ 19 \quad + 19}$	Add 19 to both sides
$4x = 24$	Focus on multiplication by 4

$$\begin{array}{r} \overline{4} \quad \overline{4} \\ x = 6 \end{array} \quad \begin{array}{l} \text{Divide both sides by 4} \\ \text{Our Solution} \end{array}$$

This leads to a 5-step process to solve any linear equation. While all five steps aren't always needed, this can serve as a guide to solving equations.

1. Distribute through any parentheses.
2. Combine like terms on each side of the equation.
3. Get the variables on one side by adding or subtracting
4. Solve the remaining 2-step equation (add or subtract then multiply or divide)
5. Check your answer by plugging it back in for x to find a true statement.

The order of these steps is very important.

World View Note: The Chinese developed a method for solving equations that involved finding each digit one at a time about 2000 years ago!

We can see each of the above five steps worked through our next example.

Example 66.

$$\begin{array}{r} 4(2x - 6) + 9 = 3(x - 7) + 8x \\ 8x - 24 + 9 = 3x - 21 + 8x \\ 8x - 15 = 11x - 21 \\ \underline{- 8x} \quad \underline{- 8x} \\ -15 = 3x - 21 \\ \underline{+ 21} \quad \underline{+ 21} \\ 6 = 3x \\ \underline{3} \quad \underline{3} \\ 2 = x \end{array} \quad \begin{array}{l} \text{Distribute 4 and 3 through parenthesis} \\ \text{Combine like terms } -24 + 9 \text{ and } 3x + 8x \\ \text{Notice the variable is on both sides} \\ \text{Subtract } 8x \text{ from both sides} \\ \text{Focus on subtraction of 21} \\ \text{Add 21 to both sides} \\ \text{Focus on multiplication by 3} \\ \text{Divide both sides by 3} \\ \text{Our Solution} \end{array}$$

Check:

$$\begin{array}{r} 4[2(2) - 6] + 9 = 3[(2) - 7] + 8(2) \\ 4[4 - 6] + 9 = 3[-5] + 8(2) \end{array} \quad \begin{array}{l} \text{Plug 2 in for each } x. \text{ Multiply inside parenthesis} \\ \text{Finish parenthesis on left, multiply on right} \end{array}$$

$4[-2] + 9 = -15 + 8(2)$	Finish multiplication on both sides
$-8 + 9 = -15 + 16$	Add
$1 = 1$	True!

When we check our solution of $x = 2$ we found a true statement, $1 = 1$. Therefore, we know our solution $x = 2$ is the correct solution for the problem.

There are two special cases that can come up as we are solving these linear equations. The first is illustrated in the next two examples. Notice we start by distributing and moving the variables all to the same side.

Example 67.

$3(2x - 5) = 6x - 15$	Distribute 3 through parenthesis
$6x - 15 = 6x - 15$	Notice the variable on both sides
$\underline{-6x} \quad \underline{-6x}$	Subtract $6x$ from both sides
$-15 = -15$	Variable is gone! True!

Here the variable subtracted out completely! We are left with a true statement, $-15 = -15$. If the variables subtract out completely and we are left with a true statement, this indicates that the equation is always true, no matter what x is. Thus, for our solution we say **all real numbers** or $\mathbb{R}$.

Example 68.

$2(3x - 5) - 4x = 2x + 7$	Distribute 2 through parenthesis
$6x - 10 - 4x = 2x + 7$	Combine like terms $6x - 4x$
$2x - 10 = 2x + 7$	Notice the variable is on both sides
$\underline{-2x} \quad \underline{-2x}$	Subtract $2x$ from both sides
$-10 \neq 7$	Variable is gone! False!

Again, the variable subtracted out completely! However, this time we are left with a false statement, this indicates that the equation is never true, no matter what x is. Thus, for our solution we say **no solution** or $\emptyset$.

1.3 Practice - General Linear Equations

Solve each equation.

1) $2 - (-3a - 8) = 1$

3) $-5(-4 + 2v) = -50$

5) $66 = 6(6 + 5x)$

7) $0 = -8(p - 5)$

9) $-2 + 2(8x - 7) = -16$

11) $-21x + 12 = -6 - 3x$

13) $-1 - 7m = -8m + 7$

15) $1 - 12r = 29 - 8r$

17) $20 - 7b = -12b + 30$

19) $-32 - 24v = 34 - 2v$

21) $-2 - 5(2 - 4m) = 33 + 5m$

23) $-4n + 11 = 2(1 - 8n) + 3n$

25) $-6v - 29 = -4v - 5(v + 1)$

27) $2(4x - 4) = -20 - 4x$

29) $-a - 5(8a - 1) = 39 - 7a$

31) $-57 = -(-p + 1) + 2(6 + 8p)$

33) $-2(m - 2) + 7(m - 8) = -67$

35) $50 = 8(7 + 7r) - (4r + 6)$

37) $-8(n - 7) + 3(3n - 3) = 41$

39) $-61 = -5(5r - 4) + 4(3r - 4)$

41) $-2(8n - 4) = 8(1 - n)$

43) $-3(-7v + 3) + 8v = 5v - 4(1 - 6v)$

45) $-7(x - 2) = -4 - 6(x - 1)$

47) $-6(8k + 4) = -8(6k + 3) - 2$

49) $-2(1 - 7p) = 8(p - 7)$

2) $2(-3n + 8) = -20$

4) $2 - 8(-4 + 3x) = 34$

6) $32 = 2 - 5(-4n + 6)$

8) $-55 = 8 + 7(k - 5)$

10) $-(3 - 5n) = 12$

12) $-3n - 27 = -27 - 3n$

14) $56p - 48 = 6p + 2$

16) $4 + 3x = -12x + 4$

18) $-16n + 12 = 39 - 7n$

20) $17 - 2x = 35 - 8x$

22) $-25 - 7x = 6(2x - 1)$

24) $-7(1 + b) = -5 - 5b$

26) $-8(8r - 2) = 3r + 16$

28) $-8n - 19 = -2(8n - 3) + 3n$

30) $-4 + 4k = 4(8k - 8)$

32) $16 = -5(1 - 6x) + 3(6x + 7)$

34) $7 = 4(n - 7) + 5(7n + 7)$

36) $-8(6 + 6x) + 4(-3 + 6x) = -12$

38) $-76 = 5(1 + 3b) + 3(3b - 3)$

40) $-6(x - 8) - 4(x - 2) = -4$

42) $-4(1 + a) = 2a - 8(5 + 3a)$

44) $-6(x - 3) + 5 = -2 - 5(x - 5)$

46) $-(n + 8) + n = -8n + 2(4n - 4)$

48) $-5(x + 7) = 4(-8x - 2)$

50) $8(-8n + 4) = 4(-7n + 8)$

Solving Linear Equations - Fractions

Objective: Solve linear equations with rational coefficients by multiplying by the least common denominator to clear the fractions.

Often when solving linear equations we will need to work with an equation with fraction coefficients. We can solve these problems as we have in the past. This is demonstrated in our next example.

Example 69.

$$\frac{3}{4}x - \frac{7}{2} = \frac{5}{6} \quad \text{Focus on subtraction}$$

$$\underline{+ \frac{7}{2} + \frac{7}{2}} \quad \text{Add } \frac{7}{2} \text{ to both sides}$$

Notice we will need to get a common denominator to add $\frac{5}{6} + \frac{7}{2}$. Notice we have a common denominator of 6. So we build up the denominator, $\frac{7}{2}\left(\frac{3}{3}\right) = \frac{21}{6}$, and we can now add the fractions:

$$\frac{3}{4}x - \frac{21}{6} = \frac{5}{6} \quad \text{Same problem, with common denominator 6}$$

$$\underline{+ \frac{21}{6} + \frac{21}{6}} \quad \text{Add } \frac{21}{6} \text{ to both sides}$$

$$\frac{3}{4}x = \frac{26}{6} \quad \text{Reduce } \frac{26}{6} \text{ to } \frac{13}{3}$$

$$\frac{3}{4}x = \frac{13}{3} \quad \text{Focus on multiplication by } \frac{3}{4}$$

We can get rid of $\frac{3}{4}$ by dividing both sides by $\frac{3}{4}$. Dividing by a fraction is the same as multiplying by the reciprocal, so we will multiply both sides by $\frac{4}{3}$.

$$\left(\frac{4}{3}\right)\frac{3}{4}x = \frac{13}{3}\left(\frac{4}{3}\right) \quad \text{Multiply by reciprocal}$$

$$x = \frac{52}{9} \quad \text{Our solution!}$$

While this process does help us arrive at the correct solution, the fractions can make the process quite difficult. This is why we have an alternate method for dealing with fractions - clearing fractions. Clearing fractions is nice as it gets rid of the fractions for the majority of the problem. We can easily clear the fractions

by finding the LCD and multiplying each term by the LCD. This is shown in the next example, the same problem as our first example, but this time we will solve by clearing fractions.

Example 70.

$$\begin{array}{rcl} \frac{3}{4}x - \frac{7}{2} = \frac{5}{6} & \text{LCD} = 12, \text{ multiply each term by 12} \\ \frac{(12)3}{4}x - \frac{(12)7}{2} = \frac{(12)5}{6} & \text{Reduce each 12 with denominators} \\ (3)3x - (6)7 = (2)5 & \text{Multiply out each term} \\ 9x - 42 = 10 & \text{Focus on subtraction by 42} \\ + \underline{42 + 42} & \text{Add 42 to both sides} \\ 9x = 52 & \text{Focus on multiplication by 9} \\ \underline{9} \quad \underline{9} & \text{Divide both sides by 9} \\ x = \frac{52}{9} & \text{Our Solution} \end{array}$$

The next example illustrates this as well. Notice the 2 isn't a fraction in the original equation, but to solve it we put the 2 over 1 to make it a fraction.

Example 71.

$$\begin{array}{rcl} \frac{2}{3}x - 2 = \frac{3}{2}x + \frac{1}{6} & \text{LCD} = 6, \text{ multiply each term by 6} \\ \frac{(6)2}{3}x - \frac{(6)2}{1} = \frac{(6)3}{2}x + \frac{(6)1}{6} & \text{Reduce 6 with each denominator} \\ (2)2x - (6)2 = (3)3x + (1)1 & \text{Multiply out each term} \\ 4x - 12 = 9x + 1 & \text{Notice variable on both sides} \\ - \underline{4x} \quad - \underline{4x} & \text{Subtract } 4x \text{ from both sides} \\ -12 = 5x + 1 & \text{Focus on addition of 1} \\ - \underline{1} \quad - \underline{1} & \text{Subtract 1 from both sides} \\ -13 = 5x & \text{Focus on multiplication of 5} \\ \underline{5} \quad \underline{5} & \text{Divide both sides by 5} \\ -\frac{13}{5} = x & \text{Our Solution} \end{array}$$

We can use this same process if there are parenthesis in the problem. We will first distribute the coefficient in front of the parenthesis, then clear the fractions. This is seen in the following example.

Example 72.

$$\begin{array}{ll} \frac{3}{2}\left(\frac{5}{9}x + \frac{4}{27}\right) = 3 & \text{Distribute } \frac{3}{2} \text{ through parenthesis, reducing if possible} \\ \frac{5}{6}x + \frac{2}{9} = 3 & \text{LCD} = 18, \text{ multiply each term by 18} \\ \frac{(18)5}{6}x + \frac{(18)2}{9} = \frac{(18)3}{9} & \text{Reduce 18 with each denominator} \\ (3)5x + (2)2 = (18)3 & \text{Multiply out each term} \\ 15x + 4 = 54 & \text{Focus on addition of 4} \\ \underline{-4 \quad -4} & \text{Subtract 4 from both sides} \\ 15x = 50 & \text{Focus on multiplication by 15} \\ \cdot \underline{15} \quad \underline{15} & \text{Divide both sides by 15. Reduce on right side.} \\ x = \frac{10}{3} & \text{Our Solution} \end{array}$$

While the problem can take many different forms, the pattern to clear the fraction is the same, after distributing through any parentheses we multiply each term by the LCD and reduce. This will give us a problem with no fractions that is much easier to solve. The following example again illustrates this process.

Example 73.

$$\begin{array}{ll} \frac{3}{4}x - \frac{1}{2} = \frac{1}{3}\left(\frac{3}{4}x + 6\right) - \frac{7}{2} & \text{Distribute } \frac{1}{3}, \text{ reduce if possible} \\ \frac{3}{4}x - \frac{1}{2} = \frac{1}{4}x + 2 - \frac{7}{2} & \text{LCD} = 4, \text{ multiply each term by 4.} \\ \frac{(4)3}{4}x - \frac{(4)1}{2} = \frac{(4)1}{4}x + \frac{(4)2}{1} - \frac{(4)7}{2} & \text{Reduce 4 with each denominator} \\ (1)3x - (2)1 = (1)1x + (4)2 - (2)7 & \text{Multiply out each term} \\ 3x - 2 = x + 8 - 14 & \text{Combine like terms } 8 - 14 \\ 3x - 2 = x - 6 & \text{Notice variable on both sides} \\ \underline{-x \quad -x} & \text{Subtract } x \text{ from both sides} \\ 2x - 2 = -6 & \text{Focus on subtraction by 2} \\ \underline{+2 \quad +2} & \text{Add 2 to both sides} \\ 2x = -4 & \text{Focus on multiplication by 2} \\ \underline{2} \quad \underline{2} & \text{Divide both sides by 2} \\ x = -2 & \text{Our Solution} \end{array}$$

World View Note: The Egyptians were among the first to study fractions and linear equations. The most famous mathematical document from Ancient Egypt is the Rhind Papyrus where the unknown variable was called “heap”

1.4 Practice - Fractions

Solve each equation.

1) $\frac{3}{5}(1+p) = \frac{21}{20}$

3) $0 = -\frac{5}{4}(x - \frac{6}{5})$

5) $\frac{3}{4} - \frac{5}{4}m = \frac{113}{24}$

7) $\frac{635}{72} = -\frac{5}{2}(-\frac{11}{4} + x)$

9) $2b + \frac{9}{5} = -\frac{11}{5}$

11) $\frac{3}{2}(\frac{7}{3}n + 1) = \frac{3}{2}$

13) $-a - \frac{5}{4}(-\frac{8}{3}a + 1) = -\frac{19}{4}$

15) $\frac{55}{6} = -\frac{5}{2}(\frac{3}{2}p - \frac{5}{3})$

17) $\frac{16}{9} = -\frac{4}{3}(-\frac{4}{3}n - \frac{4}{3})$

19) $-\frac{5}{8} = \frac{5}{4}(r - \frac{3}{2})$

21) $-\frac{11}{3} + \frac{3}{2}b = \frac{5}{2}(b - \frac{5}{3})$

23) $-(-\frac{5}{2}x - \frac{3}{2}) = -\frac{3}{2} + x$

25) $\frac{45}{16} + \frac{3}{2}n = \frac{7}{4}n - \frac{19}{16}$

27) $\frac{3}{2}(v + \frac{3}{2}) = -\frac{7}{4}v - \frac{19}{6}$

29) $\frac{47}{9} + \frac{3}{2}x = \frac{5}{3}(\frac{5}{2}x + 1)$

2) $-\frac{1}{2} = \frac{3}{2}k + \frac{3}{2}$

4) $\frac{3}{2}n - \frac{8}{3} = -\frac{29}{12}$

6) $\frac{11}{4} + \frac{3}{4}r = \frac{163}{32}$

8) $-\frac{16}{9} = -\frac{4}{3}(\frac{5}{3} + n)$

10) $\frac{3}{2} - \frac{7}{4}v = -\frac{9}{8}$

12) $\frac{41}{9} = \frac{5}{2}(x + \frac{2}{3}) - \frac{1}{3}x$

14) $\frac{1}{3}(-\frac{7}{4}k + 1) - \frac{10}{3}k = -\frac{13}{8}$

16) $-\frac{1}{2}(\frac{2}{3}x - \frac{3}{4}) - \frac{7}{2}x = -\frac{83}{24}$

18) $\frac{2}{3}(m + \frac{9}{4}) - \frac{10}{3} = -\frac{53}{18}$

20) $\frac{1}{12} = \frac{4}{3}x + \frac{5}{3}(x - \frac{7}{4})$

22) $\frac{7}{6} - \frac{4}{3}n = -\frac{3}{2}n + 2(n + \frac{3}{2})$

24) $-\frac{149}{16} - \frac{11}{3}r = -\frac{7}{4}r - \frac{5}{4}(-\frac{4}{3}r + 1)$

26) $-\frac{7}{2}(\frac{5}{3}a + \frac{1}{3}) = \frac{11}{4}a + \frac{25}{8}$

28) $-\frac{8}{3} - \frac{1}{2}x = -\frac{4}{3}x - \frac{2}{3}(-\frac{13}{4}x + 1)$

30) $\frac{1}{3}n + \frac{29}{6} = 2(\frac{4}{3}n + \frac{2}{3})$

Solving Linear Equations - Formulas

Objective: Solve linear formulas for a given variable.

Solving formulas is much like solving general linear equations. The only difference is we will have several variables in the problem and we will be attempting to solve for one specific variable. For example, we may have a formula such as $A = \pi r^2 + \pi rs$ (formula for surface area of a right circular cone) and we may be interested in solving for the variable s . This means we want to isolate the s so the equation has s on one side, and everything else on the other. So a solution might look like $s = \frac{A - \pi r^2}{\pi r}$. This second equation gives the same information as the first, they are algebraically equivalent, however, one is solved for the area, while the other is solved for s (slant height of the cone). In this section we will discuss how we can move from the first equation to the second.

When solving formulas for a variable we need to focus on the one variable we are trying to solve for, all the others are treated just like numbers. This is shown in the following example. Two parallel problems are shown, the first is a normal one-step equation, the second is a formula that we are solving for x

Example 74.

$$\begin{array}{rcl}
 3x = 12 & wx = z & \text{In both problems, } x \text{ is multiplied by something} \\
 \hline 3 & \hline w & \hline w \\
 x = 4 & x = \frac{z}{w} & \text{To isolate the } x \text{ we divide by 3 or } w. \\
 & & \text{Our Solution}
 \end{array}$$

We use the same process to solve $3x = 12$ for x as we use to solve $wx = z$ for x . Because we are solving for x we treat all the other variables the same way we would treat numbers. Thus, to get rid of the multiplication we divided by w . This same idea is seen in the following example.

Example 75.

$$\begin{array}{rcl}
 m + n = p & \text{for } n & \text{Solving for } n, \text{ treat all other variables like numbers} \\
 \hline m & \hline m & \hline m \\
 n = p - m & & \text{Subtract } m \text{ from both sides} \\
 & & \text{Our Solution}
 \end{array}$$

As p and m are not like terms, they cannot be combined. For this reason we leave the expression as $p - m$. This same one-step process can be used with grouping symbols.

Example 76.

$$\frac{a(x-y)}{(x-y)} = \frac{b}{(x-y)} \quad \begin{array}{l} \text{for } a \text{ Solving for } a, \text{ treat } (x-y) \text{ like } a \text{ number} \\ \text{Divide both sides by } (x-y) \end{array}$$

$$a = \frac{b}{x-y} \quad \text{Our Solution}$$

Because $(x - y)$ is in parenthesis, if we are not searching for what is inside the parenthesis, we can keep them together as a group and divide by that group. However, if we are searching for what is inside the parenthesis, we will have to break up the parenthesis by distributing. The following example is the same formula, but this time we will solve for x .

Example 77.

$$\begin{array}{ll} a(x-y) = b & \text{for } x \text{ Solving for } x, \text{ we need to distribute to clear parenthesis} \\ ax - ay = b & \text{This is a two-step equation, } ay \text{ is subtracted from our } x \text{ term} \\ + ay + ay & \text{Add } ay \text{ to both sides} \\ \hline ax = b + ay & \text{The } x \text{ is multiplied by } a \\ \hline \frac{ax}{a} = \frac{b+ay}{a} & \text{Divide both sides by } a \\ x = \frac{b+ay}{a} & \text{Our Solution} \end{array}$$

Be very careful as we isolate x that we do not try and cancel the a on top and bottom of the fraction. This is not allowed if there is any adding or subtracting in the fraction. There is no reducing possible in this problem, so our final reduced answer remains $x = \frac{b+ay}{a}$. The next example is another two-step problem

Example 78.

$$\begin{array}{ll} y = mx + b & \text{for } m \text{ Solving for } m, \text{ focus on addition first} \\ -b & -b \\ \hline y - b = mx & \text{Subtract } b \text{ from both sides} \\ & m \text{ is multiplied by } x. \\ \hline \frac{y-b}{x} = m & \text{Divide both sides by } x \\ & \text{Our Solution} \end{array}$$

It is important to note that we know we are done with the problem when the variable we are solving for is isolated or alone on one side of the equation and it does not appear anywhere on the other side of the equation.

The next example is also a two-step equation, it is the problem we started with at the beginning of the lesson.

Example 79.

$$\begin{array}{ll}
A = \pi r^2 + \pi r s & \text{for } s \text{ Solving for } s, \text{ focus on what is added to the term with } s \\
- \pi r^2 - \pi r^2 & \text{Subtract } \pi r^2 \text{ from both sides} \\
\hline
A - \pi r^2 = \pi r s & s \text{ is multiplied by } \pi r \\
\frac{\pi r}{\pi r} \quad \frac{\pi r}{\pi r} & \text{Divide both sides by } \pi r \\
\hline
\frac{A - \pi r^2}{\pi r} = s & \text{Our Solution}
\end{array}$$

Again, we cannot reduce the πr in the numerator and denominator because of the subtraction in the problem.

Formulas often have fractions in them and can be solved in much the same way we solved with fractions before. First identify the LCD and then multiply each term by the LCD. After we reduce there will be no more fractions in the problem so we can solve like any general equation from there.

Example 80.

$$\begin{array}{ll}
h = \frac{2m}{n} & \text{for } m \text{ To clear the fraction we use LCD } = n \\
(n)h = \frac{(n)2m}{n} & \text{Multiply each term by } n \\
\frac{nh}{2} = \frac{2m}{2} & \text{Reduce } n \text{ with denominators} \\
\frac{nh}{2} = \frac{2m}{2} & \text{Divide both sides by 2} \\
\frac{nh}{2} = m & \text{Our Solution}
\end{array}$$

The same pattern can be seen when we have several fractions in our problem.

Example 81.

$$\begin{array}{ll}
\frac{a}{b} + \frac{c}{b} = e & \text{for } a \text{ To clear the fraction we use LCD } = b \\
\frac{(b)a}{b} + \frac{(b)c}{b} = e(b) & \text{Multiply each term by } b \\
a + c = eb & \text{Reduce } b \text{ with denominators} \\
- c - c & \text{Subtract } c \text{ from both sides} \\
\hline
a = eb - c & \text{Our Solution}
\end{array}$$

Depending on the context of the problem we may find a formula that uses the same letter, one capital, one lowercase. These represent different values and we must be careful not to combine a capital variable with a lower case variable.

Example 82.

$$a = \frac{A}{2-b} \text{ for } b \text{ Use LCD } (2-b) \text{ as a group}$$

$(2 - b)a = \frac{(2 - b)A}{2 - b}$	Multiply each term by $(2 - b)$
$(2 - b)a = A$	reduce $(2 - b)$ with denominator
$2a - ab = A$	Distribute through parenthesis
$\frac{-2a}{-a} \quad \frac{-2a}{-a}$	Subtract $2a$ from both sides
$-ab = A - 2a$	The b is multiplied by $-a$
$\frac{-a}{-a} \quad \frac{-a}{-a}$	Divide both sides by $-a$
$b = \frac{A - 2a}{-a}$	Our Solution

Notice the A and a were not combined as like terms. This is because a formula will often use a capital letter and lower case letter to represent different variables. Often with formulas there is more than one way to solve for a variable. The next example solves the same problem in a slightly different manner. After clearing the denominator, we divide by a to move it to the other side, rather than distributing.

Example 83.

$a = \frac{A}{2 - b}$	for b Use LCD = $(2 - b)$ as a group
$(2 - b)a = \frac{(2 - b)A}{2 - b}$	Multiply each term by $(2 - b)$
$(2 - b)a = A$	Reduce $(2 - b)$ with denominator
$\frac{-a}{-a} \quad \frac{-a}{-a}$	Divide both sides by a
$2 - b = \frac{A}{a}$	Focus on the positive 2
$\frac{-2}{-2} \quad \frac{-2}{-2}$	Subtract 2 from both sides
$-b = \frac{A}{a} - 2$	Still need to clear the negative
$(-1)(-b) = (-1)\frac{A}{a} - 2(-1)$	Multiply (or divide) each term by -1
$b = -\frac{A}{a} + 2$	Our Solution

Both answers to the last two examples are correct, they are just written in a different form because we solved them in different ways. This is very common with formulas, there may be more than one way to solve for a variable, yet both are equivalent and correct.

World View Note: The father of algebra, Persian mathematician Muhammad ibn Musa Khwarizmi, introduced the fundamental idea of balancing by subtracting the same term to the other side of the equation. He called this process al-jabr which later became the word algebra.

1.5 Practice - Formulas

Solve each of the following equations for the indicated variable.

1) $ab = c$ for b

2) $g = \frac{h}{i}$ for h

3) $\frac{f}{g}x = b$ for x

4) $p = \frac{3y}{q}$ for y

5) $3x = \frac{a}{b}$ for x

6) $\frac{ym}{b} = \frac{c}{d}$ for y

7) $E = mc^2$ for m

8) $DS = ds$ for D

9) $V = \frac{4}{3}\pi r^3$ for π

10) $E = \frac{mv^2}{2}$ for m

11) $a + c = b$ for c

12) $x - f = g$ for x

13) $c = \frac{4y}{m+n}$ for y

14) $\frac{rs}{a-3} = k$ for r

15) $V = \frac{\pi Dn}{12}$ for D

16) $F = k(R - L)$ for k

17) $P = n(p - c)$ for n

18) $S = L + 2B$ for L

19) $T = \frac{D-d}{L}$ for D

20) $I = \frac{E_a - E_q}{R}$ for E_a

21) $L = L_o(1 + at)$ for L_o

22) $ax + b = c$ for x

23) $2m + p = 4m + q$ for m

24) $q = 6(L - p)$ for L

25) $\frac{k-m}{r} = q$ for k

26) $R = aT + b$ for T

27) $h = vt - 16t^2$ for v

28) $S = \pi rh + \pi r^2$ for h

29) $Q_1 = P(Q_2 - Q_1)$ for Q_2

30) $L = \pi(r_1 + r_2) + 2d$ for r_1

31) $R = \frac{kA(T_1 + T_2)}{d}$ for T_1

32) $P = \frac{V_1(V_2 - V_1)}{g}$ for V_2

33) $ax + b = c$ for a

34) $rt = d$ for r

35) $lwh = V$ for w

36) $V = \frac{\pi r^2 h}{3}$ for h

37) $\frac{1}{a} + b = \frac{c}{a}$ for a

38) $\frac{1}{a} + b = \frac{c}{a}$ for b

39) $at - bw = s$ for t

40) $at - bw = s$ for w

41) $ax + bx = c$ for a

42) $x + 5y = 3$ for x

43) $x + 5y = 3$ for y

44) $3x + 2y = 7$ for x

45) $3x + 2y = 7$ for y

46) $5a - 7b = 4$ for a

47) $5a - 7b = 4$ for b

48) $4x - 5y = 8$ for x

49) $4x - 5y = 8$ for y

50) $C = \frac{5}{9}(F - 32)$ for F