Factoring out the GCF and Factoring out -1

1. Factor out the greatest common factor in the expressions given.

   a) $3x-12$
   b) $16a^2b+20a^3b-12a^2b^2$
   c) $3a^2-12$
   d) $3a^3-12a^2$
   e) $20x+5x^3$
   f) $3x(x-2)+8x^3(x-2)-11(x-2)$

2. Factor out -1 from ethe given expression.

$$-5x^3+2x^2-x-8$$

3. Solve each of the following equations. Make sure to check your solution.

   a) $(x-2)(x+3)(2x+1)=0$ b) $m(m+7)=0$ c) $x^2=9x$ d) $8x^3=50x^2$

4. Find all numbers that satisfy the following condition: if we square the number, we get back the same number.

Practice Problems

1. Factor out the greatest common factor from each of the following.

   a) $10a^2b^2-15ab^3+25a^2b^3c$
   b) $6x^3-3x^2-15x^4$
   c) $a^2-a^3+a^4$
   d) $6a^2b+12a^3b-30a^3b^2$
   e) $x^5-2x^4+4x^3$
   f) $3xy(a-3)+8t(a-3)-200x^5(a-3)$

2. Factor out -1 from each of the following.

   a) x^3-x^5+2 b) $-x^2+3x-1$ c) $-x^2+3x-5$

3. Solve each of the following equations. Make sure to check your solutions.

   a) $(w+5)(w-1)=0$
   b) $x(x-2)(x+3)=0$
   c) $2(x-2)(x+3)=0$
   d) $x^2=4x$
   e) $x^2+6x=0$
   f) $3x^3=75x^2$

4. A number has the following property: if we square it, we obtain the opposite of the number. Find all such numbers.

Sample Problems - Answers

1. a) $3(x-4)$ b) $4a^2b(5a-3b+4)$ c) $3(a^2-4)$ d) $3a^2(a-4)$
   e) $5x(x^2+4)$ f) $(x-2)(8x^3+3x-11)$
2. $-(5x^3-2x^2+x+8)$
3. a) $2, -3$, and $-\frac{1}{2}$ b) 0 and -7 c) 0 and 9 d) 0 and $\frac{25}{4}$
4. $0, 1$

Practice Problems - Answers

1. a) $5ab^2(2a-3b+5abc)$ b) $3x^2(2x-5x^2-1)$ c) $a^2(a^2-a+1)$ d) $6a^2b(2a-5ab+1)$
   e) $x^3(x^2-2x+4)$ f) $(a-3)(3xy+8t-200x^5)$
2. a) $-(-x^3+x^5-2)$ b) $-(x^2-3x+1)$ c) $-(x^2-3x+5)$
3. a) $-5, 1$ b) $0, 2, -3$ c) $2, -3$ d) $0, 4$ e) $0, -6$ f) $0, 25$
4. $0, -1$

Sample Problems - Solutions

1. Completely factor each of the following.

a) $3x-12$
Solution: We start with the greatest common factor (or GCF). In this case, the GCF is 3.

$$3x-12=\boxed{3(x-4)}$$

What is in the parentheses, $x-4$ can not be further factored. We can easily check our work by multiplication.

b) $16a^2b+20a^3b-12a^2b^2$
Solution: First we identify the GCF (greatest common factor). In this case, the GCF is $4a^2b$. So we have

$$4a^2b\,(\qquad\qquad\qquad)$$

and need to figure out what to write into the parenthese so that the multiplication backwards works. What do we have to multiply $4a^2b$ by so the result is $16a^2b$? The answer is $\dfrac{16a^2b}{4a^2b}=4$, so we have so far

$$4a^2b\,(4\qquad\qquad\qquad)$$

Next, what do we have to multiply $4a^2b$ by so the result is $20a^3b$? The answer is $\dfrac{20a^3b}{4a^2b}=5a$ and so now we have

$$4a^2b\,(4+5a\qquad\qquad)$$

Next, what do we have to multiply $4a^2b$ by so the result is $-12a^2b^2$? The answer is $\dfrac{-12a^2b^2}{4a^2b}=-3b$ and so now we have

$$\boxed{4a^2b\,(4+5a-3b)}$$

We check via multiplication backwards:

$$\begin{aligned}4a^2b\,(4+5a-3b) &= 4a^2b\,(4)+4a^2b\,(5a)+4a^2b\,(-3b)\\ &= 16a^2b+20a^3b-12a^2b^2\end{aligned}$$

and so our solution is correct.

c) $3a^2-12$
Solution: The greatest common factor (or GCF) is 3.

$$3a^2-12=3\left(a^2-4\right)$$

We check our work by multiplication:

$$3\left(a^2-4\right)=3a^2-12$$

and so our answer, $\boxed{3\left(a^2-4\right)}$ is correct.

d) $3a^3-12a^2$
Solution: The greatest common factor (or GCF) is $3a^2$.

$$3a^3-12a^2=3a^2\,(\qquad\qquad)$$

what do we have to multiply $3a^2$ by so the result is $3a^3$? The answer is $\dfrac{3a^3}{3a^2}=a$ and so now we have

$$3a^3-12a^2=3a^2\,(a\qquad\qquad)$$

Next, what do we have to multiply $3a^2$ by so the result is $-12a^2$? The answer is $\dfrac{-12a^2}{3a^2} = -4$ and so now we have

$$3a^3 - 12a^2 = 3a^2(a-4)$$

We check our work by multiplication:

$$3a^2(a-4) = 3a^2 \cdot a - 3a^2 \cdot 4 = 3a^3 - 12a^2$$

and so our answer, $\boxed{3a^2(a-4)}$ is correct.

e) $20x + 5x^3$
Solution: We rearrange the terms by degree first and then factor out the GCF.

$$20x + 5x^3 = 5x^3 + 20x = 5x\left(x^2+4\right)$$

The final answer is $\boxed{5x\left(x^2+4\right)}$. We can easily check the result by mulitplication.

f) $3x(x-2) + 8x^3(x-2) - 11(x-2)$

Solution: Now the GCF is the linear expression $x-2$. So we factor ot out:

$$(x-2)(\qquad\qquad)$$

What do we have to multiply $x-2$ by so the result is $3x(x-2)$? The answer is $3x$ and so now we have

$$(x-2)(3x\qquad\qquad)$$

Next, what do we have to multiply $x-2$ by so the result is $8x^3(x-2)$? The answer is $8x^3$ and so now we have

$$(x-2)\left(3x+8x^3\qquad\qquad\right)$$

Next, what do we have to multiply $x-2$ by so the result is $-11(x-2)$? The answer is -11 and so now we have

$$(x-2)\left(3x+8x^3-11\right)$$

It is good practice to rearrange polynomials by degree and so our answer is $\boxed{(x-2)\left(8x^3+3x-11\right)}$.

2. Factor out -1 from the given expression.

$$-5x^3 + 2x^2 - x - 8$$

Solution: We start with a minus sign (short for -1) and a parentheses.

$$-5x^3 + 2x^2 - x - 8 = -(\qquad\qquad)$$

Inside the parenthese, we write the original expression, but change all signs. Again, when we multiply back to check and apply the distributive law, we should get back the original expression.

$$-5x^3 + 2x^2 - x - 8 = -\left(5x^3 - 2x^2 + x + 8\right)$$

So our answer is $\boxed{-\left(5x^3 - 2x^2 + x + 8\right)}$.

3. Solve each of the following equations. Make sure to check your solution.

a) $(x-2)(x+3)(2x+1)=0$

Solution: Since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the zero product rule. Most of these were already done for us as the right-hand side is zero and the left-hand side is completely factored. All we need to do is apply the zero product rule. **A product can only be zero if one of its factors is zero.** $(x-2)(x+3)(2x+1)=0$ means that either $x-2=0$ or $x+3=0$ or $2x+1=0$. We solve these linear equations separately:

$$\begin{array}{ccccc} x-2=0 & \text{or} & x+3=0 & \text{or} & 2x+1=0 \\ x=2 & & x=-3 & & 2x=-1 \\ & & & & x=-\dfrac{1}{2} \end{array}$$

We check all three solutions. If $x=2$, then

$$(2-2)(2+3)(2(2)+1)=0\cdot 5\cdot 5=0$$

If $x=-3$, then

$$(-3-2)(-3+3)(2(-3)+1)=-5\cdot 0\cdot(-5)=0$$

and if $x=-\dfrac{1}{2}$, then

$$\left(-\frac{1}{2}-2\right)\left(-\frac{1}{2}+3\right)\left(2\left(-\frac{1}{2}\right)+1\right)=-\frac{3}{2}\cdot\frac{5}{2}\cdot 0=0$$

and so all three numbers, $\boxed{2,\ -3,\ \text{and}\ -\frac{1}{2}}$ are correct.

b) $m(m+7)=0$

Solution: We will apply the zero product rule. **A product can only be zero if one of its factors is zero.** $m(m+7)=0$ means that either $m=0$. We solve these linear equations separately and obtain $m=0$ and $m=-7$. We check: If $m=0$, then

$$0(0+7)=0\cdot 7=0$$

and if $m=-7$, then

$$-7(-7+7)=-7\cdot 0$$

and so both numbers, $\boxed{0 \text{ and } -7}$ are correct.

c) $x^2=9x$

Solution: Since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the zero product rule.

$$\begin{aligned} x^2 &= 9x && \text{subtract } 9x \\ x^2-9x &= 0 && \text{factor out the GCF} \\ x(x-9) &= 0 \end{aligned}$$

A product can only be zero if one of its factors is zero. $x(x-9)=0$ means that either $x=0$ or $x-9=0$. We solve these linear equations separately and obtain $\boxed{0 \text{ and } 9}$. We check: $0^2=9\cdot 0$ and $9^2=9\cdot 9$ and so our solution is correct.

d) $8x^3 = 50x^2$
Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the zero product rule.

$$\begin{aligned} 8x^3 &= 50x^2 && \text{subtract } 50x^2 \\ 8x^3 - 50x^2 &= 0 && \text{the GCF is } 2x^2 \\ 2x^2(4x-25) &= 0 \end{aligned}$$

We now apply the zero product rule. If this product is zero, then either $2x^2 = 0$ or $4x - 25 = 0$. We solve these equations for x.

$$\begin{aligned} 2x^2 &= 0 && \text{or} && 4x - 25 = 0 \\ 2 \cdot x \cdot x &= 0 && \text{or} && 4x = 25 \\ x &= 0 && \text{or} && x = \frac{25}{4} \end{aligned}$$

We check both solutions. If $x = 0$, then

$$\text{LHS} = 8 \cdot 0^3 = 8 \cdot 0 = 0 \quad \text{and} \quad \text{RHS} = 50 \cdot 0^2 = 50 \cdot 0 = 0$$

If $x = \frac{25}{4}$, then

$$\begin{aligned} \text{LHS} &= 8\left(\frac{25}{4}\right)^3 = \frac{8}{1} \cdot \frac{15\,625}{64} = \frac{15\,625}{8} \\ \text{RHS} &= 50\left(\frac{25}{4}\right)^2 = \frac{50}{1} \cdot \frac{625}{16} = \frac{15\,625}{8} \end{aligned}$$

Thus both solutions, $\boxed{0 \text{ and } \frac{25}{4}}$ are correct.

4. Find all numbers that satisfy the following condition: if we square the number, we get back the same number.
Solution: Let us denote the number by x. The equation is

$$\begin{aligned} x^2 &= x && \text{reduce one side to zero} \\ x^2 - x &= 0 && \text{factor} \\ x(x-1) &= 0 && \text{apply the zero property} \end{aligned}$$

$$\begin{aligned} x &= 0 && \text{or} && x - 1 = 0 \\ x &= 0 && \text{or} && x = 1 \end{aligned}$$

Thus there are two numbers, 0 and 1, satisfying the property. We check: $0^2 = 0$ and $1^2 = 1$. Thus our answer is: $\boxed{0 \text{ and } 1}$.