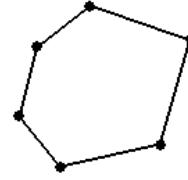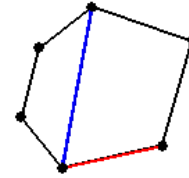## Part 1 - Definitions

Definition: An $n-$sided **polygon** is n many points on a plane, connected with straight lines, as shown.

In every polygon, the number of points (also called vertices) and the number of sides are equal. Polygons with four sides are also called **quadrilaterals**.

Definition: A line segment connecting two 'neighboring' points is a **side** of the polygon. A line segment connecting two points, if it is not a side, then we call it a **diagonal**.

On the picture, the red line segment is a side, and the blue line segment is a diagonal of the polygon.

**Definition:** A **rectangle** is a four-sided polygon with four right angles.

We also refer to four-sided polygons as **quadrilateral**s. Although they are not part of the definition of a rectangle, there are a few important properties of rectangles that we should keep in mind. **In every rectangle, the opposite sides have equal lengths. The two diagonals of any rectangle are equally long and they bisect each other.** (To bisect something means to cut it into two equal parts.)
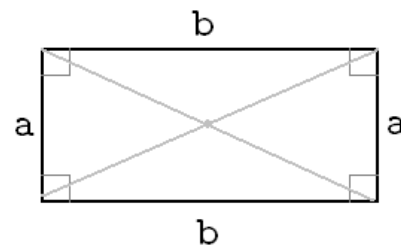
Enrichment

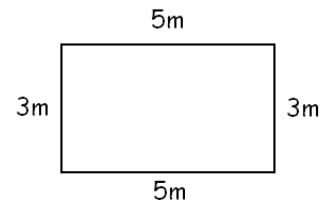Draw and count the diagonals in each of the polygons shown.
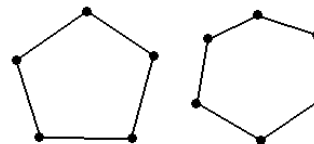
Part 2 - Perimeter

Definition: The **perimeter** of any geometric object is the length of its boundary.

In general, we can always think of the perimeter as a fencing problem. If we have a property, how long of a fence do we need to buy to completely fence around the property? We will denote perimeter by P. Perimeter is a *length*, we measure it in meters (m), centimeters (cm), inches (in), feet (ft), kilometers (km), or miles (mi).

Example 1. Compute the perimeter of a rectangle with sides 3 meters and 5 meters long.

Solution: If we think fencing, we mentally walk around a rectangle-shaped property to figure out how much fencing to buy. That is the same as simply adding the lengths of all four sides sides. The lengths of only two sides were given, but this should not be a problem. The

opposite sides of a rectangle are equally long. Thus we can compute the perimeter as

$$P = 3\text{m} + 5\text{m} + 3\text{m} + 5\text{m} = 16\text{m}$$

So the perimeter of this rectangle is $\boxed{P = 16\text{m}}$.

Example 1v. Find the perimeter and area of the rectangle with sides 11ft and 9ft long.

Solution - Youtube link

In general, the perimeter of a quadrilateral with sides a, b, c, and d is $P = a+b+c+d$. In case of a rectangle, the opposite sides are equally long, so $c = a$ and $d = b$ and this makes the perimeter formula simpler.

**Theorem:** The perimeter of a rectangle with sides a and b is $P = 2a + 2b$.

This means that in our previous example, the perimeter of a 3m by 5m rectangle is

$$P = 2\,(3\text{m}) + 2\,(5\text{m}) = 6\text{m} + 10\text{m} = 16\text{m}$$

Example 2. Find the perimeter of the object shown. Angles that look like right angles are right angles.
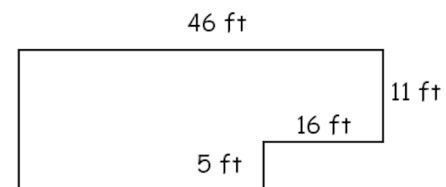
Solution: It is much easier to discuss geometry if we label points and sides. Consider the picture shown. We are clearly missing the lengths of some sides for the perimeter, so we need to figure out those lengths first.
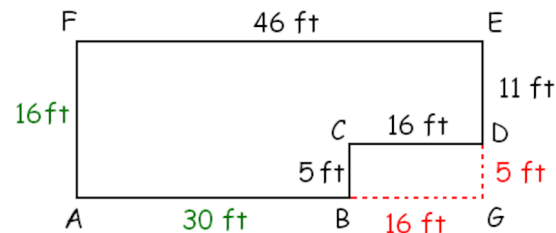
We first draw line AB beyond point B and line ED beyond point D as shown. These lines intersect each other in point G. Since $BGDC$ is a rectangle, its opposite sides are equally long. Therefore, $BG = 16\text{ft}$ and $DG = 5\text{ft}$.

The quadrilateral $AGEF$ is also a rectangle, and so FA is the same length as EG.

$FA = 16$ft, because $FA = EG = ED + DG = 11$ft $+ 5$ft $= 16$ft. Also, AG is as long as FE.

$$
\begin{aligned}
AG &= FE \\
AB + BG &= 46\text{ft} \qquad \text{we know } BG = 16\text{ft} \\
AB + 16\text{ft} &= 46\text{ft} \qquad \text{subtract } 16\text{ft} \\
AB &= 30\text{ft}
\end{aligned}
$$

We can now compute the perimeter, starting from point A, and moving counterclockwise.

$$P = AB + BC + CD + DE + EF + FA = 30\text{ft} + 5\text{ft} + 16\text{ft} + 11\text{ft} + 46\text{ft} + 16\text{ft} = \boxed{124\text{ft}}$$

Part 3 - Area

The **area** of a geometric object is a measurement of its surface.

Understanding and remembering area formulas are easier if we know how they were derived. While we could think about perimeter as a fencing problem, area can be thought of as follows. Suppose a geometric object is a room. How many tiles do we need to buy to cover the entire room? Of course, we have to first agree on the size of the tiles we use to measure area.

Definition: The **area** of a 1ft by 1ft square (shown on the picture) is defined to be 1ft^2 (square-foot). Similar definitions can be formulated with mi^2, cm^2, in^2, etc. The area of any object, measured in ft^2, is the number of these 1ft by 1ft square tiles needed to cover the object, cutting and pasting allowed.

1 ft

1 ft

$A = 1\ \text{ft}^2$
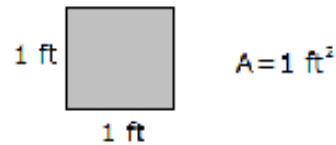
Area is not a length like perimeter. Area is measured in square-meters (m^2), square-centimeters (cm^2), square-inches (in^2), square-feet (ft^2), square-kilometers (km^2), or square-miles (mi^2). etc., and is usually denoted by A.
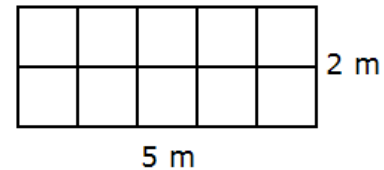
Example 3. Find the area of the figure shown on the picture.

Solution: We simply count the tiles we need to cover this object. Since the figure can be covered using five unit tiles, its area is $\boxed{A = 5\text{ft}^2}$.

1 ft   1 ft   1 ft

1 ft

1 ft

1 ft
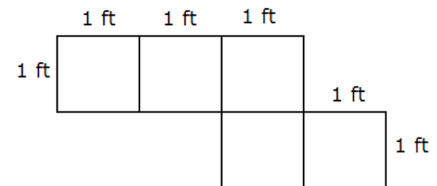
Example 4. Find the area of the figure shown on the picture.

Solution: Since the figure can be covered using ten 1m by 1m squares, its area is $\boxed{A = 10\text{m}^2}$.

Theorem: The area of a rectangle with sides x and y is $A = xy$.

Proof: We will not formally prove this theorem. Instead, we will just informally argue for this formula. The main idea should be clear from the previous example. Consider a rectangle with sides 3m and 5m. The area of this rectangle will be as many square-meters as many 1m by 1 m square tiles are needed to cover it.
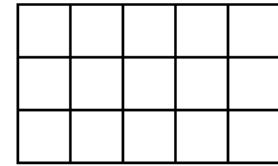
Once we place a grid on the rectangle, it is easy to see how many such squares are needed. The rectangle is composed of five columns of squares, where each column consists of three squares. Thus we split the rectangle into fifteen unit tiles, and so the area is 15m^2. This shows that as long as the lengths of the sides are integers, we can place a grid on it, and the number of unit square tiles is the product of the length of the two sides.

In reality, this theorem is very difficult to prove, because not all side lengths happen to be integers. Mathematicians proved that this formula is true even if the sides of the rectangle are not integers.

Another interesting fact is that logically, we counted how many meter^2 we have. The computation for the area, however, is slightly different with regards to units. Instead of counting meter^2, we literally multiply meter by meter.

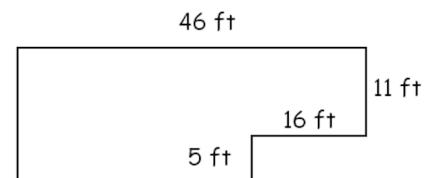$$A = xy = 3\text{m}\,(5\text{m}) = 15\text{m}^2 \quad \text{and not} \quad 15 \cdot 1\text{m}^2$$

Area computation will always yield for the right unit.

Example 5. Find the area of a rectangle with sides 13 in and 7 in.

Solution: We apply the formula.
$$A = xy = 13\text{in}\,(7\text{in}) = \boxed{91\text{in}^2}$$

Example 6. Find the area of the object shown. Angles that look like right angles are right angles.

**Solution:** To compute the area, we have two options.

Method 1: We can think of the region as the sum of two rectangles: $CDEF$ is a rectangle with sides 11ft by 16ft, and $ABFG$ is a rectangle with sides 30ft by 16ft. So the area is

$$
\begin{aligned}
A &= A_{CDEF} + A_{ABFG} \\
&= 11\text{ft} \cdot 16\text{ft} + 16\text{ft} \cdot 30\text{ft} \\
&= 176\text{ft}^2 + 480\text{ft}^2 = \boxed{656\text{ft}^2}
\end{aligned}
$$

Method 2: We can also think of the region as a big rectangle from which a corner was removed.

The big (red) rectangle, $AHEG$ has sides 16ft and 46ft long. The smaller (blue) rectangle has sides 5ft by 16ft. So the area is the difference.

$$
\begin{aligned}
A &= A_{AHEG} - A_{BHDC} \\
&= 46\text{ft} \cdot 16\text{ft} - 16\text{ft} \cdot 5\text{ft} \\
&= 736\text{ft}^2 - 80\text{ft}^2 = \boxed{656\text{ft}^2}
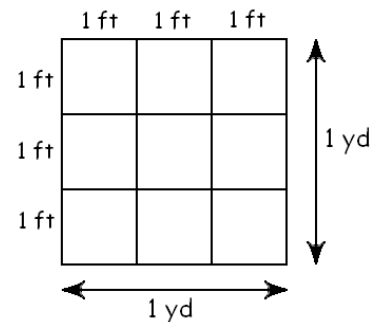\end{aligned}
$$
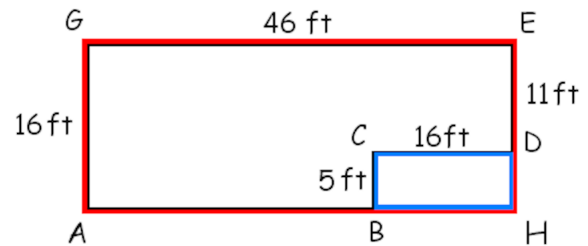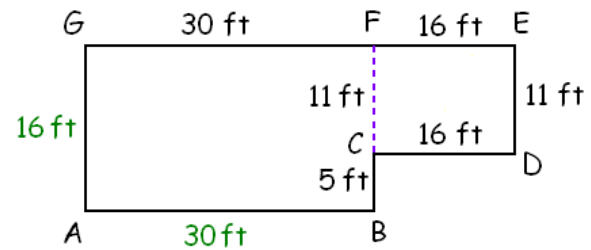
Example 7. Find the perimeter and area of the figure shown. Solution - Youtube link
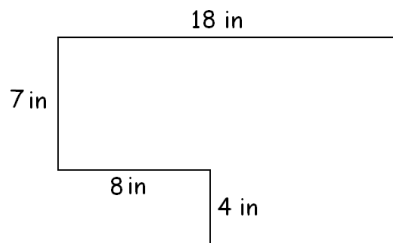
Discussion:

One yard equals to three feet. Consider the picture below and discuss: how many square-feet is one square-yard? Can you show the same result algebraically?

1. Compute the perimeter and area of a rectangle with sides 12cm by 8cm.

2. Consider the figure shown. Angles that look like right angles are right anges.

    a) Compute the perimeter of the figure. Include units in your answer.

   b) Compute the area of the figure. Include units in your answer.
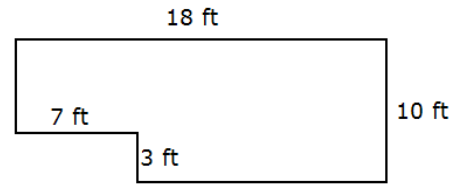
3. Consider the figure shown. Angles that look like right angles are right anges.

    a) Compute the perimeter of the figure. Include units in your answer.

    b) Compute the area of the figure. Include units in your answer.
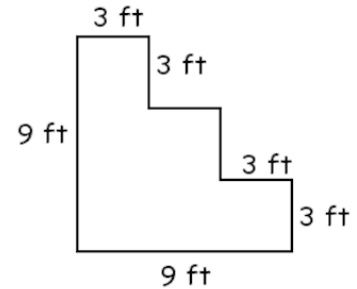
 Enrichment

The rectangle shown on the picture has sides 2cm and 3cm long. The four other shapes are all squares with sides 1cm. Which region has a greater area, the blue or the yellow?
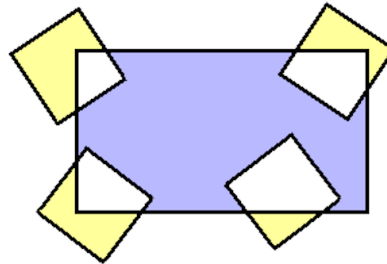
Answers

Discussion

$1\,\text{yd}^2 = 9\text{ft}^2$

Algebraically: if $1\,\text{yd} = 3\text{ft}$, then $\quad 1\,\text{yd}^2 = 1\,\text{yd} \cdot 1\,\text{yd} = 3\text{ft} \cdot 3\text{ft} = 9\text{ft}^2$

Practice Problems

1. $P = 40\text{cm}, \; A = 96\text{cm}^2$ 2. $P = 56\text{ft}, \; A = 159\text{ft}^2$ 3. $P = 36\text{ft}, \; A = 54\text{ft}^2$