

Definition: A function f is **one-to-one (or injective)** if for all a and b in its domain, if $a \neq b$, then $f(a) \neq f(b)$.

Alternative definition: A function f is **one-to-one (or injective)** if for all a and b in its domain, if $f(a) = f(b)$, then $a = b$.

Definition: A function f is **increasing** on an interval I if for all a and b in I , if $a < b$, then $f(a) \leq f(b)$.

Definition: A function f is **strictly increasing** on an interval I if for all a and b in I , if $a < b$, then $f(a) < f(b)$.

Definition: A function f is **decreasing** on an interval I if for all a and b in I , if $a < b$, then $f(a) \geq f(b)$.

Definition: A function f is **strictly decreasing** on an interval I if for all a and b in I , if $a < b$, then $f(a) > f(b)$.

The concept of a **continuous function** is very important. Although this term will not be precisely defined, the intuitive idea of a continuous function is that we can draw its graph without lifting the pencil.

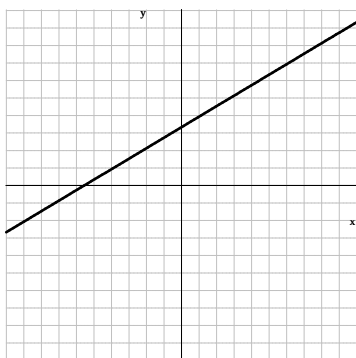
For example, $f(x) = x^2$ is a continuous function but $g(x) = \frac{1}{x}$ is not; it is not continuous at $x = 0$.

Basic Functions

1.) Linear functions $f(x) = mx + b$ where $m \neq 0$.

The graph is a straight line. One useful form of the equation can be obtained by factoring out the slope:

$$f(x) = mx + b = m \left(x + \frac{b}{m} \right)$$



$$m > 0$$

Case 1. If $m > 0$

domain: $\mathbb{R}$

range: $\mathbb{R}$

y -intercept: $(0, b)$

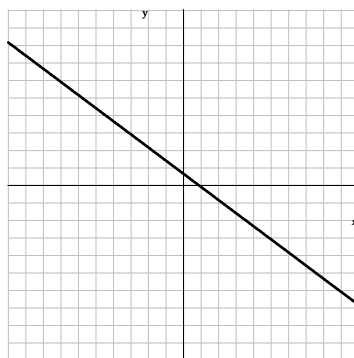
x -intercept: $\left(-\frac{b}{m}, 0\right)$

one-to-one

no maximum or minimum

strictly increasing

continuous on $\mathbb{R}$



$$m < 0$$

Case 2. If $m < 0$

domain: $\mathbb{R}$

range: $\mathbb{R}$

y -intercept: $(0, b)$

x -intercept: $\left(-\frac{b}{m}, 0\right)$

one-to-one

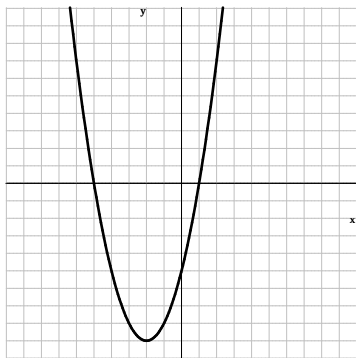
no maximum or minimum

strictly decreasing

continuous on $\mathbb{R}$

2.) Quadratic functions $f(x) = ax^2 + bx + c$ where $a \neq 0$.

The graph is a parabola. It opens upward if $a > 0$ and opens downward if $a < 0$.



$$a > 0$$

Case 1. If $a > 0$

Example: $f(x) = x^2 + 4x - 5$

standard form: $f(x) = (x + 2)^2 - 9$

factored form: $f(x) = (x + 5)(x - 1)$

domain: $\mathbb{R}$ range: $[-9, \infty)$

y -intercept: $(0, -5)$

x -intercepts: $(-5, 0)$ and $(1, 0)$

not one-to-one

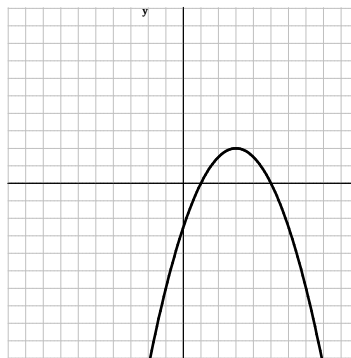
no maximum

minimum: $(-2, -9)$

strictly decreasing on $(-\infty, -2)$

strictly increasing on $(-2, \infty)$

continuous on $\mathbb{R}$



$$a < 0$$

Case 2. If $a < 0$

Example: $f(x) = -\frac{1}{2}x^2 + 3x - \frac{5}{2}$

standard form: $f(x) = -\frac{1}{2}(x - 3)^2 + 2$

factored form: $f(x) = -\frac{1}{2}(x - 1)(x - 5)$

domain: $\mathbb{R}$ range: $(-\infty, 2]$

y -intercept: $(0, -\frac{5}{2})$

x -intercepts: $(1, 0)$ and $(5, 0)$

not one-to-one

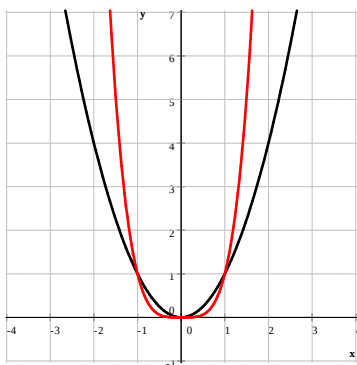
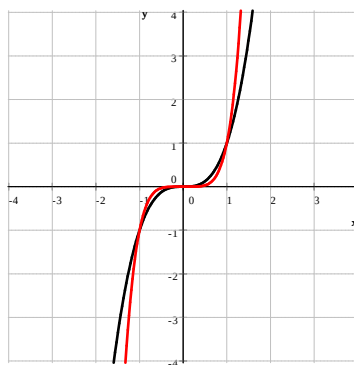
no minimum

maximum: $(3, 2)$

strictly increasing on $(-\infty, 3)$

strictly decreasing on $(3, \infty)$

continuous on $\mathbb{R}$

3.) Monomials $f(x) = x^n$  n is even n is oddCase 1. If n is evendomain: $\mathbb{R}$ range: $[0, \infty)$ y -intercept: $(0, 0)$ x -intercept: $(0, 0)$

not one-to-one

no maximum

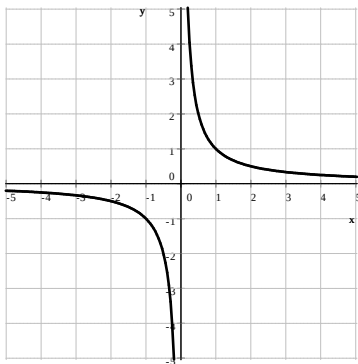
minimum: $(0, 0)$ strictly decreasing on $(-\infty, 0)$ strictly increasing on $(0, \infty)$ continuous on $\mathbb{R}$ black graph: $f(x) = x^2$ red graph: $f(x) = x^4$ Case 2. If n is odddomain: $\mathbb{R}$ range: $\mathbb{R}$ y -intercept: $(0, 0)$ x -intercept: $(0, 0)$

one-to-one

no minimum or maximum

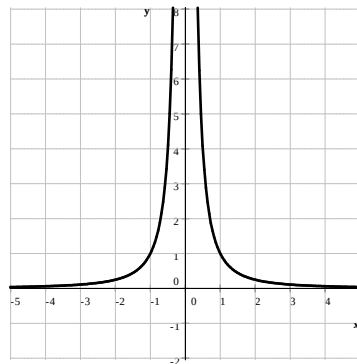
strictly increasing on $\mathbb{R}$ continuous on $\mathbb{R}$ black graph: $f(x) = x^3$ red graph: $f(x) = x^5$

4.) The rational functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$



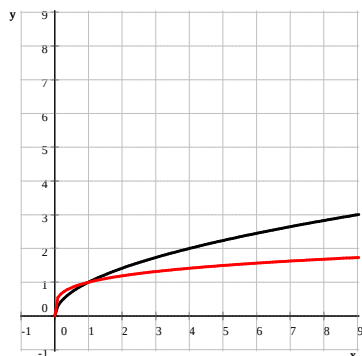
$$f(x) = \frac{1}{x}$$

domain: $\mathbb{R} \setminus \{0\}$ range: $\mathbb{R} \setminus \{0\}$
 no y -intercept
 no x -intercept
 one-to-one
 no maximum or minimum
 strictly decreasing on $(-\infty, 0)$ and on $(0, \infty)$
 not continuous at $x = 0$
 vertical asymptote: the line $x = 0$
 horizontal asymptote: the line $y = 0$



$$g(x) = \frac{1}{x^2}$$

domain: $\mathbb{R} \setminus \{0\}$ range: $(0, \infty)$
 no y -intercept
 no x -intercept
 not one-to-one
 no minimum or maximum
 strictly increasing on $(-\infty, 0)$ and
 strictly decreasing on $(0, \infty)$
 not continuous at $x = 0$
 vertical asymptote: the line $x = 0$
 horizontal asymptote: the line $y = 0$

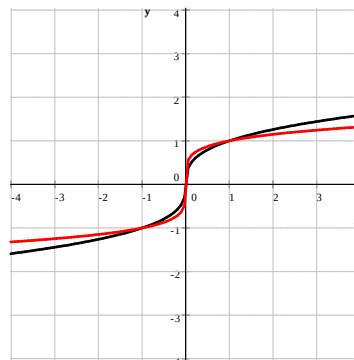
5.) Radical functions $f(x) = \sqrt[n]{x}$  n is evenCase 1. If n is evendomain: $[0, \infty)$ range: $[0, \infty)$ y -intercept: $(0, 0)$ x -intercept: $(0, 0)$

one-to-one

no maximum

minimum: $(0, 0)$

strictly increasing

continuous on $(0, \infty)$ black graph: $f(x) = \sqrt{x}$ red graph: $f(x) = \sqrt[4]{x}$  n is oddCase 2. If n is odddomain: $\mathbb{R}$ range: $\mathbb{R}$ y -intercept: $(0, 0)$ x -intercept: $(0, 0)$

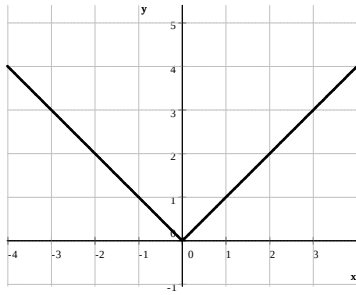
one-to-one

no minimum or maximum

strictly increasing

continuous on $\mathbb{R}$ black graph: $f(x) = \sqrt[3]{x}$ red graph: $f(x) = \sqrt[5]{x}$

6.) Absolute value function, $f(x) = |x|$



domain: $\mathbb{R}$ range: $[0, \infty)$

x -intercept: $(0, 0)$, y -intercept: $(0, 0)$

not one-to-one

no maximum

minimum: $(0, 0)$

strictly decreasing on $(-\infty, 0)$ and strictly increasing on $(0, \infty)$

continuous on $\mathbb{R}$

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