Sample Problems

1. How many gallons of 3% acid solution must be mixed with 60 gallons of 10% acid solution to obtain an acid solution that is 8%?
2. How many gallons of each of a 4% and an 11% salt solutions should be mixed to obtain 35 gallons of a 7% solution?
3. How much water should we add to 20 gallons of 15% acid solution to dilute it to a concentration of 12%?

Practice Problems

1. How many liters of a 15% acid solution should be mixed with 10 liters of a 36% acid solution to obtain a mixture that is 20%?
2. How many liters of a 17% acid solution should be mixed with 8 liters of an 11% acid solution to obtain a mixture that is 15%?
3. How many gallons of each of a 7% and a 23% salt solutions should be mixed to obtain 32 gallons of a 12% solution?
4. How many liters of each of a 12% and a 33% alcohol solutions should be mixed to obtain 21 liters of a 25% solution?
5. How much water should we add to 5 gallons of 18% acid solution to dilute it to a concentration of 10%?
6. How much water should we add to 10 gallons of 30% acid solution to dilute it to a concentration of 25%?
7. How much pure alcohol should we add to 6 gallons of 10% acid solution to obtain a solution that is 40%?
8. How much pure alcohol should we add to 18 gallons of 35% acid solution to obtain a solution that is 55%?

Sample Problems - Answers

1. How many gallons of 3% acid solution must be mixed with 60 gallons of 10% acid solution to obtain an acid solution that is 8%? 24 gallons of 3% solution with 60 gallons of 10% solution
2. How many gallons of each of a 4% and an 11% salt solutions should be mixed to obtain 35 gallons of a 7% solution? 20 gallons of 4% solution with 15 gallons of 11% solution
3. How much water should we add to 20 gallons of 15% acid solution to dilute it to a concentration of 12%? 5 gallons

Practice Problems - Answers

1. How many liters of a 15% acid solution should be mixed with 10 liters of a 36% acid solution to obtain a mixture that is 20%? 32 liters
2. How many liters of a 17% acid solution should be mixed with 8 liters of an 11% acid solution to obtain a mixture that is 15%? 16 liters
3. How many gallons of each of a 7% and a 23% salt solutions should be mixed to obtain 32 gallons of a 12% solution? 10 gallons of 23% solution with 22 gallons of 7% solution
4. How many liters of each of a 12% and a 33% alcohol solutions should be mixed to obtain 21 liters of a 25% solution? 13 liters of 33% solution with 8 liters of 12% solution
5. How much water should we add to 5 gallons of 18% acid solution to dilute it to a concentration of 10%? 4 gallons
6. How much water should we add to 10 gallons of 30% acid solution to dilute it to a concentration of 25%? 2 gallons
7. How much pure alcohol should we add to 6 gallons of 10% acid solution to obtain a solution that is 40%? 3 gallons
8. How much pure alcohol should we add to 18 gallons of 35% acid solution to obtain a solution that is 55%? 8 gallons

Sample Problems - SOLUTIONS

1. How many gallons of 3% acid solution must be mixed with 60 gallons of 10% acid solution to obtain an acid solution that is 8%? 24 gallons of 3% solution with 60 gallons of 10% solution
Solution: Let us denote the amount of 3% solution we use. Then

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0.03	$0.03x$
Component 2	60	0.1	$60\,(0.1)$
Mixture	$x+60$	0.08	$0.08\,(x+60) = 0.03x + 60\,(0.1)$

We obtain the equation by stating that the amount of solvant in the components must add up to the amount of solvant. (In other words, the last entry in the third row can be written in two different ways: the product of $x+60$ and 8%; and the sum of $0.03x$ and $60\,(0.1)$

$$\begin{aligned}
0.08\,(x+60) &= 0.03x + 60\,(0.1) \\
0.08\,(x+60) &= 0.03x + 6 && \text{multiply by 100 to make numbers 'nice'} \\
8\,(x+60) &= 3x + 600 && \text{distribute} \\
8x + 480 &= 3x + 600 && \text{subtract } 3x \\
5x + 480 &= 600 && \text{subtract 480} \\
5x &= 120 && \text{divide by 5} \\
x &= 24
\end{aligned}$$

Thus we need to mix 24 gallons of 3% solution with 60 gallons of 10% solution.
We check: suppose we mix the two solutions specified above. We need to find how much solution and how much solvant we have, hoping that the amount of solvant indeed will be 8% of the amount of mixture.

	Amount of Solution	Percentage	Amount of Solvant
Component 1	60 gallons of	10% solution	$0.1\,(60) = 6$ gallons
Component 2	24 gallons of	3% solution	$0.03\,(24) = 0.72$ gallons
	$\Downarrow$		$\Downarrow$
Mixture	84 gallons		6.72 gallons

8% of 84 is $0.08\,(84) = 6.72$. Thus our solution has the right concentration.

2. How many gallons of each of a 4% and an 11% salt solutions should be mixed to obtain 35 gallons of a 7% solution? 20 gallons of 4% solution with 15 gallons of 11% solution
Solution: We will present two methods: the first one is to apply linear eqations; the second one is to apply a system of two linear equations.
Method 1 (Application of Equations) Let us denote by x the amount of 4% solution. Since we need to mix 35 gallons of a mixture, the amount of 11% solution must be $35 - x$ gallons.

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0.04	$0.04x$
Component 2	$35 - x$	0.11	$0.11\,(35 - x)$
Mixture	35	0.07	

We obtain the equation by stating that the amount of solvant in the components must add up to the amount of solvant. (In other words, the last entry in the third row can be written in two different ways: the product of 35 and 7%; and the sum of $0.04x$ and $0.11\,(35-x)$

$$\begin{aligned} 0.07\,(35) &= 0.04x + 0.11\,(35-x) \\ 2.45 &= 0.04x + 0.11\,(35-x) && \text{multiply by 100 to make numbers 'nice'} \\ 245 &= 4x + 11\,(35-x) && \text{distribute} \\ 245 &= 4x + 385 - 11x && \text{combine like terms} \\ 245 &= -7x + 385 && \text{subtract 385} \\ -140 &= -7x && \text{divide by } -7 \\ x &= 20 \end{aligned}$$

If $x = 20$, then the other amount, denoted by $35 - x$ must be $35 - 20 = 15$. Thus we need to mix 20 gallons of 4% solution with 15 gallons of 11% solution.
We check: suppose we mix the two solutions specified above. We need to find how much solution and how much solvant we have, hoping that the amount of solvant indeed will be 8% of the amount of mixture.

	Amount of Solution	Percentage	Amount of Solvant
Component 1	20 gallons	of 4% solution	$0.04\,(20) = 0.8$ gallons
Component 2	15 gallons	of 11% solution	$0.11\,(15) = 1.65$ gallons
	$\Downarrow$		$\Downarrow$
	35 gallons		$0.8 + 1.65 = 2.45$ gallons

7% of 35 is $0.07\,(35) = 2.45$ Thus our solution has the right concentration.

Method 2 (Application of System of Equations) Let us denote by x the amount of 4% solution, and by y the amount of 11% solution. Then

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0.04	$0.04x$
Component 2	y	0.11	$0.11y$
Mixture	$x + y = 35$	0.07	$0.07\,(35) = 2.45$

We need to obtain two equations:

$$\begin{aligned} x + y &= 35 \\ 0.04x + 0.11y &= 2.45 \end{aligned}$$

We will solve the system by elimination. To make numbers nicer, first we multiply both sides of the second equation by 100.

$$\begin{aligned} x + y &= 35 \\ 4x + 11y &= 245 \end{aligned}$$

To eliminate x, we will multiply both sides of the first equation by -4 and then add the two equations.

$$\begin{aligned} -4x - 4y &= -140 \\ 4x + 11y &= 245 && \text{add the two equations} \\ 7y &= 105 && \text{divide by 7} \\ y &= 15 \end{aligned}$$

Now that we know the value of y, we find x using the simplest equation:

$$\begin{aligned} x+y &= 35 && \text{we know } y=15 \\ x+15 &= 35 && \text{subtract } 15 \\ x &= 20 \end{aligned}$$

Thus we need to mix 20 gallons of 4% solution with 15 gallons of 11% solution. We check the same way as in Method1.

3. How much water should we add to 20 gallons of 15% acid solution to dilute it to a concentration of 12%? 5 gallons
Solution: The trick is to think of pure water as a 0% solution. The rest of the problem goes as the previous problems. Let us denote the amount of 3% solution we use. Then

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0	0
Component 2	20	0.15	$20\,(0.15)=3$
Mixture	$x+20$	0.12	$0.12\,(x+20)=20\,(0.15)$

$$\begin{aligned} 0.12\,(x+20) &= 20\,(0.15) \\ 0.12\,(x+20) &= 3 && \text{multiply by 100 to make numbers 'nice'} \\ 12\,(x+20) &= 300 && \text{distribute} \\ 12x+240 &= 300 && \text{subtract } 240 \\ 12x &= 60 && \text{divide by } 5 \\ x &= 5 \end{aligned}$$

Thus we need to mix 5 gallons of water with 20 gallons of 15% solution. We check:

	Amount of Solution	Percentage	Amount of Solvant
Component 1	20 gallons of	15% solution	$0.15\,(20)=3$ gallons
Component 2	5 gallons of	0% solution	0 gallons
	$\Downarrow$		$\Downarrow$
Mixture	25 gallons		3 gallons

12% of 25 is $0.12\,(25)=3$ Thus our solution has the right concentration.