Definition: A function f is **one-to-one (or injective)** if for all a and b in its domain, if $a \neq b$, then $f(a) \neq f(b)$.

Alternative definition: A function f is **one-to-one (or injective)** if for all a and b in its domain, if $f(a) = f(b)$, then $a = b$.

Definition: A function f is **increasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) \leq f(b)$.

Definition: A function f is **strictly increasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) < f(b)$.

Definition: A function f is **decreasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) \geq f(b)$.

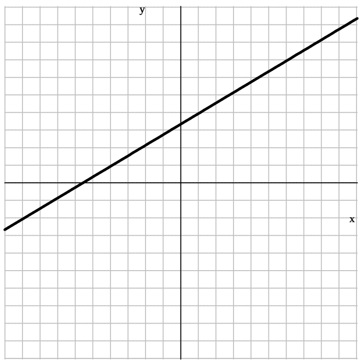
Definition: A function f is **strictly decreasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) > f(b)$.

The concept of a **continuous function** is very important. Although this term will not be precisely defined, the intuitive idea of a continuous function is thatwe can draw its graph without lifting the pencil. For example, $f(x) = x^2$ is a continuous function but $g(x) = \dfrac{1}{x}$ is not; it is not continuous at $x = 0$.

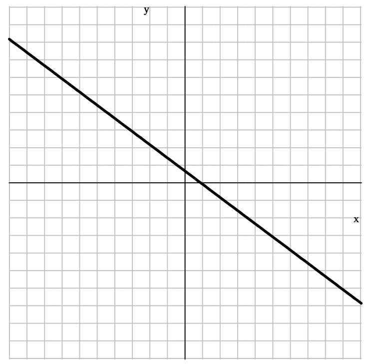
Basic Functions

1.) Linear functions $f(x) = mx + b$ where $m \neq 0$.

The graph is a straight line. One useful form of the equation can be obtained by factoring out the slope: $f(x) = mx + b = m\left(x + \dfrac{b}{m}\right)$



$m > 0$



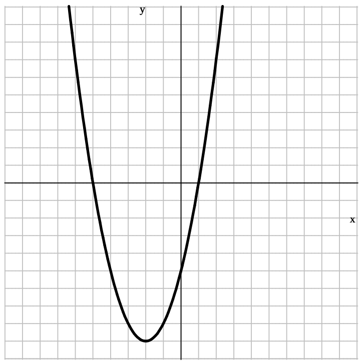
$m < 0$

Case 1. If $m > 0$
domain: $\mathbb{R}$
range: $\mathbb{R}$
$y-$intercept: $(0, b)$
$x-$intercept: $\left(-\dfrac{b}{m}, 0\right)$
one-to-one
no maximum or minimum
strictly increasing
continuous on $\mathbb{R}$

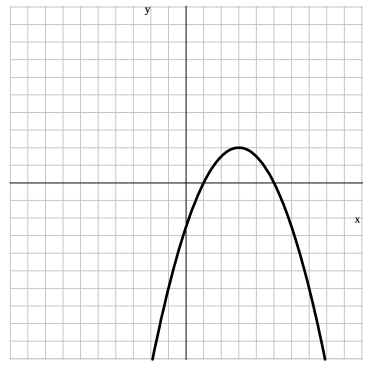
Case 2. If $m < 0$
domain: $\mathbb{R}$
range: $\mathbb{R}$
$y-$intercept: $(0, b)$
$x-$intercept: $\left(-\dfrac{b}{m}, 0\right)$
one-to-one
no maximum or minimum
strictly decreasing
continuous on $\mathbb{R}$

2.) Quadratic functions $f(x) = ax^2 + bx + c$ where $a \neq 0$.

The graph is a parabola. It opens upward if $a > 0$ and opens downward if $a < 0$.



$a > 0$



$a < 0$

Case 1. If $a > 0$

Example: $f(x) = x^2 + 4x - 5$

standard form: $f(x) = (x+2)^2 - 9$

factored form: $f(x) = (x+5)(x-1)$

domain: $\mathbb{R}$ range: $[-9, \infty)$

$y-$intercept: $(0, -5)$

$x-$intercepts: $(-5, 0)$ and $(1, 0)$

not one-to-one
no maximum
minimum: $(-2, -9)$
strictly decreasing on $(-\infty, -2)$
strictly increasing on $(-2, \infty)$
continuous on $\mathbb{R}$

Case 2. If $a < 0$

Example: $f(x) = -\dfrac{1}{2}x^2 + 3x - \dfrac{5}{2}$

standard form: $f(x) = -\dfrac{1}{2}(x-3)^2 + 2$

factored form: $f(x) = -\dfrac{1}{2}(x-1)(x-5)$

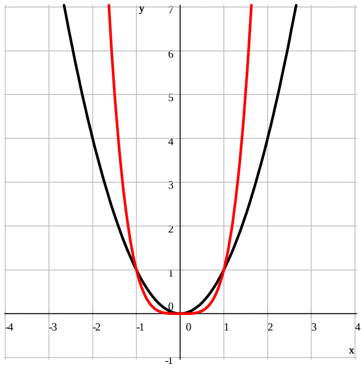
domain: $\mathbb{R}$ range: $(-\infty, 2]$

$y-$intercept: $\left(0, -\dfrac{5}{2}\right)$

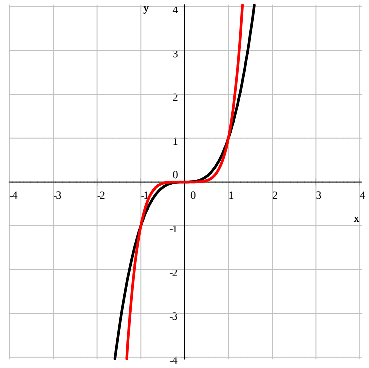
$x-$intercepts: $(1, 0)$ and $(5, 0)$

not one-to-one
no minimum
maximum: $(3, 2)$
strictly increasing on $(-\infty, 3)$
strictly decreasing on $(3, \infty)$
continuous on $\mathbb{R}$

3.) Monomials $f(x) = x^n$



n is even



n is odd

Case 1. If n is even

domain: $\mathbb{R}$ range: $[0, \infty)$

$y-$intercept: $(0, 0)$

$x-$intercept: $(0, 0)$

not one-to-one
no maximum
minimum: $(0, 0)$
strictly decreasing on $(-\infty, 0)$
strictly increasing on $(0, \infty)$
continuous on $\mathbb{R}$

black graph: $f(x) = x^2$
red graph: $f(x) = x^4$

Case 2. If n is odd

domain: $\mathbb{R}$ range: $\mathbb{R}$

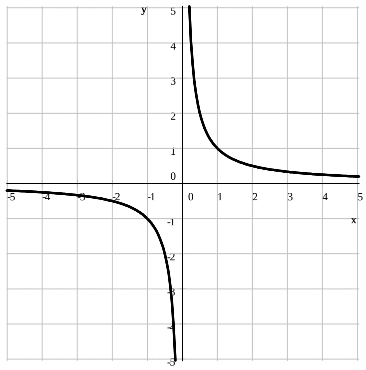
$y-$intercept: $(0, 0)$

$x-$intercept: $(0, 0)$

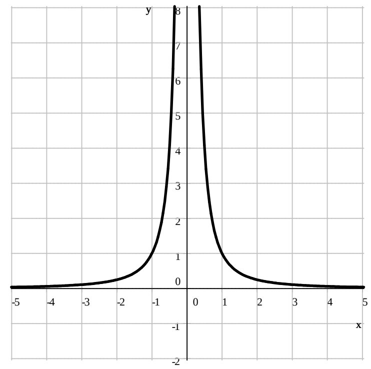
one-to-one
no minimum or maximum
strictly increasing on $\mathbb{R}$
continuous on $\mathbb{R}$

black graph: $f(x) = x^3$
red graph: $f(x) = x^5$

4.) The rational functions $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{x^2}$



$f(x) = \dfrac{1}{x}$



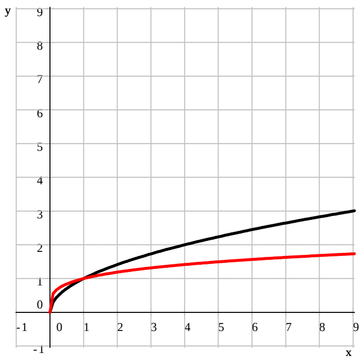
$g(x) = \dfrac{1}{x^2}$

domain: $\mathbb{R} \setminus \{0\}$ range: $\mathbb{R} \setminus \{0\}$
no $y-$intercept
no $x-$intercept
one-to-one
no maximum or minimum
strictly decreasing on $(-\infty, 0)$ and on $(0, \infty)$

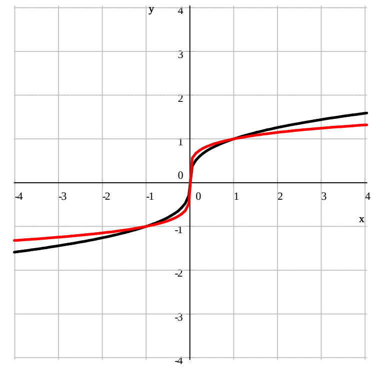
not continuous at $x = 0$
vertical asymptote: the line $x = 0$
horizontal asymptote: the line $y = 0$

domain: $\mathbb{R} \setminus \{0\}$ range: $(0, \infty)$
no $y-$intercept
no $x-$intercept
not one-to-one
no minimum or maximum
strictly increasing on $(-\infty, 0)$ and
strictly decreasing on $(0, \infty)$
not continuous at $x = 0$
vertical asymptote: the line $x = 0$
horizontal asymptote: the line $y = 0$

5.) Radical functions $f(x) = \sqrt[n]{x}$



n is even



n is odd

Case 1. If n is even

domain: $[0, \infty)$ range: $[0, \infty)$

$y-$intercept: $(0, 0)$

$x-$intercept: $(0, 0)$

one-to-one
no maximum
minimum: $(0, 0)$
strictly increasing
continuous on $(0, \infty)$

black graph: $f(x) = \sqrt{x}$
red graph: $f(x) = \sqrt[4]{x}$

Case 2. If n is odd

domain: $\mathbb{R}$ range: $\mathbb{R}$

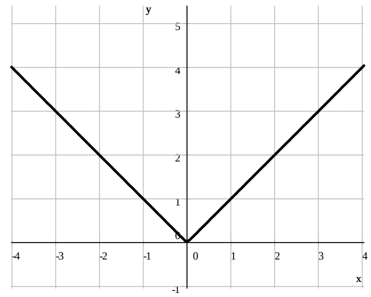
$y-$intercept: $(0, 0)$

$x-$intercept: $(0, 0)$

one-to-one
no minimum or maximum
strictly increasing
continuous on $\mathbb{R}$

black graph: $f(x) = \sqrt[3]{x}$
red graph: $f(x) = \sqrt[5]{x}$

6.) Absolute value function, $f(x) = |x|$



domain: $\mathbb{R}$ range: $[0, \infty)$

$x-$intercept: $(0, 0)$, $y-$intercept: $(0, 0)$

not one-to-one

no maximum

minimum: $(0, 0)$

strictly decreasing on $(\infty, 0)$ and strictly increasing on $(0, \infty)$

continuous on $\mathbb{R}$

