Sample Problems

Basic Fraction Problems

1. Find $\frac{2}{9}$ of 450.
2. 75 is what fraction of 400?
3. $\frac{4}{11}$ of a number is 36. Find this number.

Basic Percent Problems

4. Find 15% of 400.
5. 21 is what percent of 350?
6. 24% of what number is 72?
7. Find 16% of 3600.
8. 27 is what percent of 18?
9. 120% of what number is 150?
10. What do we get if we increase 600 by 150%?

More Linear Applications

11. Small ones weigh 3 lb, big ones weigh 4 lb. The number of small ones is 3 more than twice the number of big ones. All together, they weigh 79 lb. How many small ones are there?
12. One side of a rectangle is 7 cm shorter than five times the other side. Find the length of the sides if the perimeter of the rectangle is 118 cm.
13. The sum of two consecutive even integers is -170. Find these numbers.
14. We have a jar of coins, all quarters and dimes. All together, they are worth $17.60 We have 13 more quarters than dimes. How many quarters, how many dimes?
15. The sum of three consecutive odd integers is 57. Find these numbers.
16. One side of a rectangle is 3 cm shorter than four times the other side. Find the sides if the perimeter of the rectangle is 204 cm.
17. The opposite of a number is 18 more than twice the number. Find this number.
18. Two times a number is 5 less than the sum of 80 and the opposite of the number. Find this number.
19. Red pens cost $1 each, blue ones cost $1.50 each. We baught some pens. The number of red pens is 7 less than five times the number of blue pens. How many of each did we buy if we paid $58?
20. 55 people showed up on the party. There were 3 less women than men. How many men were there?
21. Ann and Betty dine together. The total bill is $38. Ann paid $2 more than Betty. How much did Betty pay?

Quadratic Applications

22. The product of two numbers is 640. Their difference is 12. Find these numbers.
23. One side of a rectangle is 12 ft longer than three times another side. Find the sides if the area is 288 ft^2.
24. We throw an object upward from the top of a 1200 ft tall building. The vertical position of the object, (measured in feet) t seconds after we threw it is

$$y = -16t^2 + 160t + 1200$$

a) Where is the object 3 seconds after we threw it?
b) How long does it take for the object to hit the ground?

Practice Problems

Basic Fraction Problems

1. Compute $\frac{3}{7}$ of 420.
2. 24 is what fraction of 54?
3. $\frac{3}{8}$ of a number is 60. Find this number.

Basic Percent Problems

4. Find 60% of 150.
5. 48 is what percent of 150?
6. 4% of what number is 28?
7. Find 25% of 360.
8. 32 is what percent of 80?
9. 18% of what number is 54?
10. What do we get if we increase 150 by 30%?
11. Compute 130% of 150.

More Linear Applications

12. Mary bought four less than three times the number of books that Jose did. Together they bought sixteen books. How many did Jose buy?
13. A school purchases tickets to a show. A child ticket costs \$8 and an adult ticket costs \$14. The school has paid a total of \$610 for tickets. The number of child tickets was 5 greater than three times the number of adult ticket. How many of the tickets were for adults?
14. Julia is 5 years younger than her brother, Tom. How old are they if the sum of their ages is 43?
15. One side of a rectangle is 6 in shorter than twice the other side. Find the sides of the rectangle if its perimeter is 120 in.
16. The largest angle in a triangle is three times as large as the smallest angle. The middle angle is 35° larger than the smallest angle. Find the angles in the triangle.
17. What a great ceremony! We had 150% more guests this year than last year. If the number of guests this year is 1875, how many guests were there last year?
18. The sum of two numbers is 27. Their difference is 11. Find these numbers.
19. The sum of two numbers is 11. Their difference is 27. Find these numbers.
20. The sum of two numbers is -11. Their difference is 27. Find these numbers.
21. The sum of two consecutive odd integers is 92. Find these numbers.
22. The sum of five times a number and -10 is 8 less than six times the sum of 7 and the opposite of the number. Find this number.
23. Lisa took 5 exams. The first 4 received scores of 72, 93, 86, and 82. How much did she score on the fifth exam if her average score is 74 points?

Quadratic Applications

24. The product of two numbers is 65. Their difference is 8. Find these numbers.
25. If we square a number, we get six times the number. Find all numbers with this property.
26. If we raise a number to the third power, we get four times the number. Find all numbers with this property.
27. The product of two consecutive even integers is 840. Find these numbers.
28. The area of a rectangle is 1260 m^2. Find the dimensions of the rectangle if we know that one side is 48 m longer than three times the other side.
29. We are standing on the top of a 1680 ft tall building and throw a small object upwards. At every second, we measure the distance of the object from the ground. Exactly t seconds after we threw the object, its vertical position, (measured in feet) is

$$y = -16t^2 + 256t + 1680$$

a) Find the object's position 3 seconds after we threw it.

b) How much does the object travel during the two seconds between 5 seconds and 7 seconds?

c) How long does it take for the object to reach a height of 2640 ft?

d) How long does it take for the object to hit the ground?

Answers - Sample Problems

Basic Fraction Problems

1. 100   2. $\frac{3}{16}$ 3. 99

Basic Percent Problems

4. 60   5. 6%   6. 300   7. 576   8. 150%   9. 125   10. 1500

More Linear Applications

11. 7 big, 17 small   12. 11 cm by 48 cm   13. -86 and -84 14. 41 dimes and 54 quarters

15. 17, 19, and 21   16. 21 cm by 81 cm   17. -6 18. 25   19. 10 blue and 43 red pens

20. 26 women and 29 men   21. Betty paid $18 and Ann paid $20

Quadratic Applications

22. $-32, -20$ and $20, 32$ 23. 8 ft by 3612 ft   24. a) $f(3) = 1536$ ft   b) 15 seconds

Answers - Practice Problems

Basic Fraction Problems

1. 180   2. $\frac{4}{9}$ 3. 160

Basic Percent Problems

4. 90   5. 32%   6. 700   7. 90   8. 40%   9. 300   10. 195   11. 195

More Linear Applications

12. 5   13. 15   14. Julia is 19 and Tom is 24   15. 22 in and 38 in   16. $29°, 64°, 87°$

17. 750   18. 8 and 19   19. -8 and 19   20. -19 and 8   21. 45 and 47   22. 4   23. 37

Quadratic Applications

24. 5 with 13 and -13 with -5 25. $0, 6$ 26. $-2, 0, 2$ 27. $28, 30$ and $-30, -28$

28. 14 m by 90 m   29. a) 2304 ft   b) 128 ft   c) 6 seconds and 10 seconds   d) 21 seconds

Sample Problems Solutions

Basic Fraction Problems

Consider the following problem: Find $\dfrac{3}{5}$ of 100.

Solution: $\dfrac{1}{5}$ of 100 can be obtained by splitting 100 into 5 equal shares. If we think of money, this is an easy task: 5 twenty-dollar bill make up a hundred dollar bill. In short, $\dfrac{1}{5}$ of 100 is 20.

We need $\dfrac{3}{5}$ of 100, which means that we need to take 3 shares out of the 5. This is $3(20) = 60$. Thus, $\dfrac{3}{5}$ of 100 is 60.

Notice that we obtain the same result if we simply multiply $\dfrac{3}{5}$ by 100.

$$\frac{3}{5} \cdot 100 = \frac{3}{5} \cdot \frac{100}{1} = \frac{300}{5} = 60$$

We will establish a languague we will use to solve word problems involving fractions. Consider the statement

$$\frac{3}{5} \text{ of } 100 \text{ is } 60$$

These three quantities are always present in word problems involving fractions. The following definitions are not very elegant but seem to be useful. The fraction $\dfrac{3}{5}$ will be called F for fraction. The quantity we are splitting up (in this case, 100) will be called the (of) number, since the word of is always near it. The result will be called the (is) number, since the word is is always near it. Then all word problems involving fractions.can be solved using one formula,

$$\boxed{(\text{is}) = (\text{Fraction}) \cdot (\text{of})}$$

All word problems involving fractions involve two quantities given, and we have to find the third one. Consequently, there are three types of word problems involving fractions. We will call a problem type 1 if we have to find the (is) number, type 2 if we have to find F (the fraction), and type 3 if we have to find the (of) number.

1. (Type 1) Find $\dfrac{2}{9}$ of 450.

   Solution: The formula we use is $\boxed{(\text{is}) = (\text{Fraction}) \cdot (\text{of})}$. First we identify which two quantities are given. We will denote the third one by x.

   $(\text{is}) = x$
   $F = \dfrac{2}{9}$
   $(\text{of}) = 450$

   We will substitute these into the formula and solve for x. The fraction and the (of) are given, and so (is) =F$\cdot$ (of) becomes

   $$x = \frac{2}{9} \cdot 450 = \frac{900}{9} = 100$$

   The answer is 100.

2. (Type 2) 75 is what fraction of 400?

   Solution: The formula we use is $\boxed{(\text{is}) = (\text{Fraction}) \cdot (\text{of})}$ We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

(is) = 75
$F =$ x
(of) = 400

We will substitute these into the formula and solve for x.

$$\begin{aligned} (\text{is}) &= \text{F} \cdot (\text{of}) \\ 75 &= x \cdot 400 && \text{Solve for } x, \text{ divide by } 400 \\ \frac{75}{400} &= x && \text{simplify the result} \\ x &= \frac{3}{16} \end{aligned}$$

The answer is $\frac{3}{16}$. We check: is it true that $\frac{3}{16}$ of 400 is 75? Since $\frac{3}{16}(400) = 75$, our solution is correct.

3. (Type 3) $\frac{4}{11}$ of a number is 36. Find this number.
Solution: The formula we use is $\boxed{(\text{is}) = (\text{Fraction}) \cdot (\text{of})}$ We first identify the two numbers given and call the missing number x.
(is) = 36
$F = \frac{4}{11}$
(of) = x

The fraction and the (is) are given, and so we label (of) $= x$

$$\begin{aligned} (\text{is}) &= (\text{Fraction}) \cdot (\text{of}) \\ 36 &= \frac{4}{11} \cdot x && \text{Solve for } x, \text{ divide by } \frac{4}{11} \\ \frac{36}{\left(\frac{4}{11}\right)} &= x && 36 \cdot \frac{11}{4} = 99 \\ 99 &= x \end{aligned}$$

The answer is 99. We check: is it true that $\frac{4}{11}$ of 99 is 36? Since $\frac{4}{11}(99) = 36$, our solution is correct.

We can now solve percent problems as well. All we need to know that percents are fractions with denominator 100.

Basic Percent Problems

4. (Type 1) Find 15% of 400.
Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,
(is) = x
$F = \frac{15}{100}$
(of) = 400

We will substitute these into the formula and solve for x.

$$\begin{aligned} (\text{is}) &= \text{F} \cdot (\text{of}) \\ x &= \frac{15}{100} \cdot 400 \\ x &= 60 \end{aligned}$$

Thus 15% of 400 is 60.

5. (Type 2) 21 is what percent of 350?
Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

(is) = 21
$F =$ x
(of) = 350

We will substitute these into the formula and solve for x.

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ 21 &= x \cdot 350 \qquad \text{divide by } 350 \\ \frac{21}{350} &= x \end{aligned}$$

We obtained the value of x, but not as a percent. We will convert x into a percent. First we simplify

$$x = \frac{21}{350} = \frac{\cancel{7} \cdot 3}{\cancel{7} \cdot 50} = \frac{3}{50} = \frac{3 \cdot 2}{50 \cdot 2} = \frac{6}{100} = 6\%$$

Thus 21 is 6% of 350. We can check by computing 6% of 350:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ \text{(is)} &= \frac{6}{100} \cdot 350 = 21 \end{aligned}$$

Thus our solution, 6% is correct.

6. (Type 3) 24% of what number is 72?
Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

(is) = 72
$F =$ $\frac{24}{100}$
(of) = x

We will substitute these into the formula and solve for x.

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ 72 &= \frac{24}{100} \cdot x \qquad \text{divide by } \frac{24}{100} \\ \frac{72}{\frac{24}{100}} &= x \\ 300 &= x \end{aligned}$$

The computation is $72 \div \frac{24}{100} = \frac{72}{1} \cdot \frac{100}{24} = \frac{3 \cdot 24}{1} \cdot \frac{100}{24} = \frac{300}{1} = 300$. Thus 72 is 24% of 300.
We can check by computing 24% of 300:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ \text{(is)} &= \frac{24}{100} \cdot 300 = 72 \end{aligned}$$

Thus our solution is correct. We have focused on the steps of solving such equations. Computations can be simplified by reducing fractions or working with decimals instead of fractions. We will work out three more basic problems to demonstrate these.

7. Find 16% of 3600.
Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

(is) =	x
$F =$	0.16
(of) =	3600

We will substitute these into the formula and solve for x. $0.16 \cdot 360 = 57.6$

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ x &= 0.16 \cdot 3600 \\ x &= 576 \end{aligned}$$

Thus 16% of 3600 is 576.

8. 27 is what percent of 18?
Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. Because the is-number is smaller than the of-number, we should expect a percentage larger than 100%.

(is) =	27
$F =$	x
(of) =	18

We will substitute these into the formula and solve for x.

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ 27 &= x \cdot 18 \qquad \text{divide by 18} \\ \frac{27}{18} &= x \\ x &= 1.5 = 150\% \end{aligned}$$

Thus 27 is 150% of 18. We can check by computing 150% of 18:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ \text{(is)} &= 1.5 \cdot 18 = 27 \end{aligned}$$

Thus our solution, 150% is correct.

9. 120% of what number is 150?
Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

(is) =	150
$F =$	1.2
(of) =	x

We will substitute these into the formula and solve for x.

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ 150 &= 1.2 \cdot x \qquad \text{divide by 1.2} \\ \frac{150}{1.2} &= x \\ 125 &= x \end{aligned}$$

We can check by computing 120% of 125:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ \text{(is)} &= 1.2 \cdot 125 = 150 \end{aligned}$$

Thus our solution, 125 is correct.
The following examples all boil down to one of the basic problems shown above. One advice: before starting computations, re-write the problem to a basic question. Once we have this question, the problem is easy to solve.

10. What do we get if we increase 600 by 150%?

Solution 1: We compute 150% of 600 and then we add it to 600. To compute 150% of 600 is a type 1 problem:

(is) = x
$F = \dfrac{150}{100} = 1.5$
(of) = 600

Now we use our formula to find x.

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ x &= 1.5 \cdot 600 = 900 \end{aligned}$$

So after the increase, we have $600 + 900 = 1500$.

Solution 2: This is a neat shortcut that will become very important in other problems. If a quantity is increased by 150%, then it "grew up" from 100% of itself to $100\% + 150\% = 250\%$ of itself. We can find the answer quickly if we simply compute 250% of 600. The basic question is: What is 250% of 600?(Type 1)

(is) = x
$F = 250\% = 2.5$
(of) = 600

We substitute the data into the formula:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ x &= 2.5 \cdot 600 \\ x &= 1500 \end{aligned}$$

Thus the answer is 1500.

More Applications

11. Small ones weigh 3 lb, big ones weigh 4 lb. The number of small ones is 3 more than twice the number of big ones. All together, they weigh 79 lb. How many small ones are there?

Solution: Let us denote the number of big ones by x. Then the number of small ones is $2x + 3$. We obtain the equation expressing the total weight:

$$\begin{aligned} 3(2x+3) + 4x &= 79 && \text{distribute} \\ 6x + 9 + 4x &= 79 && \text{combine like terms} \\ 10x + 9 &= 79 && \text{subtract 9} \\ 10x &= 70 && \text{divide by 10} \\ x &= 7 \end{aligned}$$

The number of big ones is then 7, and so the number of small ones is $2(7) + 3 = 17$. We check: the number of small ones, 17 is indeed 3 more than twice the number of big ones, 7. The total weight is $7(4) + 17(3) = 28 + 51 = 79$. Thus the solution is 7 big, 17 small.

12. One side of a rectangle is 7 cm shorter than five times the other side. Find the length of the sides if the perimeter of the rectangle is 118 cm.

 Solution: Let us denote the shorter side by x. Then the longer side is $5x-7$. We obtain the equation for the perimeter:

$$\begin{aligned} 2x+2(5x-7) &= 118 && \text{distribute} \\ 2x+10x-14 &= 118 && \text{combine like terms} \\ 12x-14 &= 118 && \text{add 14} \\ 12x &= 132 && \text{divide by 12} \\ x &= 11 \end{aligned}$$

 Thus the shorter side is 11 cm, the longer side is $5(11\,\text{cm})-7\,\text{cm}=48\,\text{cm}$. We check: the perimeter is $2(11\,\text{cm})+2(48\,\text{cm})=118$ cm and 48 is indeed 7 shorter than five times 11. Thus the solution is: 11 cm by 48 cm.

13. The sum of two consecutive even integers is -170. Find these numbers.

 Solution: Let us denote the smaller number by x. Then the larger number is $x+2$. The equation expresses the sum of the numbers.

$$\begin{aligned} x+x+2 &= -170 && \text{combine like terms} \\ 2x+2 &= -170 && \text{subtract 2} \\ 2x &= -172 && \text{divide by 2} \\ x &= -86 \end{aligned}$$

 Then the larger number must be $-86+2=-84$. Thus the numbers are are -86 and -84.

14. We have a jar of coins, all quarters and dimes. All together, they are worth \$17.60 We have 13 more quarters than dimes. How many quarters, how many dimes?

 Solution: Let us denote the number of dimes by x. Then the number of quarters must be $x+13$. We obtain the equation by expressing the total value, in pennies:

$$\begin{aligned} 10x+25(x+13) &= 1760 && \text{distribute} \\ 10x+25x+325 &= 1760 && \text{combine like terms} \\ 35x+325 &= 1760 && \text{subtract 325} \\ 35x &= 1435 && \text{divide by 35} \\ x &= 41 \end{aligned}$$

 Thus we have 41 dimes and $41+13=54$ quarters. We check: $41(0.10)+54(0.25)=4.10+13.50=17.6$. Thus the solution is 41 dimes and 54 quarters.

15. The sum of three consecutive odd integers is 57. Find these numbers.

 Solution: Let us denote the smallest number by x. Then the other two numbers must be $x+2$ and $x+4$. The equation expresses the sum of the three numbers.

$$\begin{aligned} x+x+2+x+4 &= 57 && \text{combine like terms} \\ 3x+6 &= 57 && \text{subtract 6} \\ x &= 51 && \text{divide by 3} \\ x &= 17 \end{aligned}$$

 Thus the three numbers are 17, and $17+2=19$, and $17+4=21$. We check: indeed, $17+19+21=57$. Thus the solution is 17, 19, and 21.

16. One side of a rectangle is 3 cm shorter than four times the other side. Find the sides if the perimeter of the rectangle is 204 cm.

    Solution: Let us denote the shorter side by x. Then the longer side is $4x-3$. We obtain the equation for the perimeter:

$$\begin{aligned} 2x+2(4x-3) &= 204 && \text{distribute} \\ 2x+8x-6 &= 204 && \text{combine like terms} \\ 10x-6 &= 204 && \text{add 6} \\ 10x &= 210 && \text{divide by 10} \\ x &= 21 \end{aligned}$$

    Thus the shorter side is 21 cm, the longer side is $4(21)-3=81$ cm. We check: the perimeter is $2(21)+2(81)=42+162=204$ cm and 81 is indeed 3 shorter than four times 21. Thus the solution is: 21 cm by 81 cm

17. The opposite of a number is 18 more than twice the number. Find this number.

    Solution: Let us denote the number by x. The the two things that we are comparing are:

the opposite of the number is	$-x$
twice the number is	$2x$

    Now we make these two equal by adding the difference to the SMALLER number. Since twice the number is 18 less than the opposite of the number, they will be equal once we add 18 to the smaller one.

$$\begin{aligned} -x &= 2x+18 && \text{add } x \\ 0 &= 3x+18 && \text{subtract 18} \\ -18 &= 3x && \text{divide by 3} \\ -6 &= x \end{aligned}$$

    Thus the number is -6. Indeed, twice -6 is -12 which is 18 less than 6, the opposite of -6. Thus the number is -6.

18. Two times a number is 5 less than the sum of 80 and the opposite of the number. Find this number.

    Solution: let us denote the number by x. The two things we are comparing:

two times a number :	$2x$
the sum of 80 and the opposite of the number :	$80+(-x)=80-x$

    We make these two equal by adding the difference to the smaller one:

$$\begin{aligned} 2x+5 &= 80-x && \text{add } x \\ 3x+5 &= 80 && \text{subtract 5} \\ 3x &= 75 && \text{divide by 3} \\ x &= 25 \end{aligned}$$

    Thus the number is 25. We check: twice 25 is 50 and the sum of 80 and the opposite of 25 is 55. 55 is indeed 5 more than 50. Thus the solution is: the number is 25.

19. Red pens cost \$1 each, blue ones cost \$1.50 each. We bought some pens. The number of red pens is 7 less than five times the number of blue pens. How many of each did we buy if we paid \$58?

    Solution: Let us denote the number of blue pens by x. Then the number of red pens is $5x-7$. The equation will express the total cost of the pens:

$$\begin{aligned} 1(5x-7)+1.50(x) &= 58 && \text{distribute} \\ 5x-7+1.5x &= 58 && \text{combine like terms} \\ 6.5x-7 &= 58 && \text{add 7} \\ 6.5x &= 65 && \text{divide by 6.5} \\ x &= 10 \end{aligned}$$

    Thus we baught 10 blue and $5(10)-7=43$ red pens. We check:

$$\begin{aligned} 43 &= 5(10)-7 \\ 1(43)+1.50(10) &= 43+15=58 \end{aligned}$$

    Thus our solution is correct; we bought 10 blue and 43 red pens.

20. 55 people showed up on the party. There were 3 less women than men. How many men were there?

    Solution: Let us denote the number of women by x. Then $x+3$ men showed up. The equation expresses the number of people:

$$\begin{aligned} x+x+3 &= 55 && \text{combine like terms} \\ 2x+3 &= 55 && \text{subtract 3} \\ 2x &= 52 && \text{divide by 2} \\ x &= 26 \end{aligned}$$

    Thus there were 26 women and 29 men on the party.

21. Ann and Betty dine together. The total bill is \$38. Ann paid \$2 more than Betty. How much did Betty pay?

    Solution: Let us denote by x the amount that Betty paid. Then Ann paid $x+2$. The equation expresses the total amount paid:

$$\begin{aligned} x+x+2 &= 38 && \text{combine like terms} \\ 2x+2 &= 38 && \text{subtract 2} \\ 2x &= 36 && \text{divide by 2} \\ x &= 18 \end{aligned}$$

    Thus Betty paid \$18 and Ann paid \$20.

Quadratic Applications

22. The product of two numbers is 640. Their difference is 12. Find these numbers.
    Solution: Let us label the smaller number as x. Then the larger number is $x+12$. The equation is

$$\begin{aligned} x(x+12) &= 640 && \text{Solve for } x \\ x^2+12x &= 640 \\ x^2+12x-640 &= 0 \end{aligned}$$

We will solve this quadratic equation by completing the square. Half of the linear coefficient is 6.

$$\begin{aligned} x^2+12x-640 &= 0 \qquad\qquad (x+6)^2 = x^2+12x+36 \\ \underbrace{x^2+12x+36}-36-640 &= 0 \\ (x+6)^2-676 &= 0 \qquad\qquad \sqrt{676}=26 \\ (x+6)^2-26^2 &= 0 \\ (x+6+26)(x+6-26) &= 0 \\ (x+32)(x-20) &= 0 \quad \Longrightarrow \quad x_1=-32 \text{ and } x_2=20 \end{aligned}$$

If $x=-32$, then the larger number is $-32+12=-20$. If $x=20$, then the larger number is $20+12=32$. **The two solutions of the equation do not determine a pair of numbers: they are the smaller numbers in two pairs!** The answer is: 20 with 32 and -32 with -20. We check in both cases: with 20 and 32

$$20(32)=640 \quad \text{and} \quad 32-20=12$$

and with -32 and -20

$$-32(-20)=640 \quad \text{and} \quad -20-(-32)=12$$

23. One side of a rectangle is 12 ft longer than three times another side. Find the sides if the area is 288 ft^2.

Solution: Let us denote the shorter side by x. Then the longer side is $3x+12$. The equation expresses the area.

$$\begin{aligned} x(3x+12) &= 288 \qquad \text{solve for } x \\ 3x^2+12x &= 288 \\ 3x^2+12x-288 &= 0 \end{aligned}$$

Since this is a quadratic equation, our only method of solving it is by the zero product rule. We will factor the expression $2x^2-3x-209$ by completing the square. First we have to factor out the leading coefficient. (For more on this, see the handout on completing the square part 3.)

$$\begin{aligned} 3x^2+12x-288 &= 0 \qquad\qquad \text{factor out 2} \\ 3\left(x^2+4x-96\right) &= 0 \end{aligned}$$

Inside the parentheses, half of the linear coefficient is 2 and so our complete square is

$$(x+2)^2 = x^2+4x+4$$

Thus we smuggle in 4.

$$\begin{aligned} 3\left(x^2+4x+4-4-96\right) &= 0 \\ 3\left((x+2)^2-100\right) &= 0 \\ 3\left((x+2)^2-10^2\right) &= 0 \\ 3(x+2+10)(x+2-10) &= 0 \\ 2(x+12)(x-8) &= 0 \quad \Longrightarrow \quad x_1=-12 \text{ and } x_2=8 \end{aligned}$$

Since x denotes the side of a rectangle, which is a distance, and **distances are never negative**, the first solution, $x_1=-12$ is immediately ruled out. If $x=8$, the other side must be $3(8)+12=36$. Thus the sides of the rectangle are 8 ft and 36 ft. We check:

$$\begin{aligned} 3(8)+12 &= 36 \checkmark \quad \text{and} \\ 8(36) &= 288 \checkmark \end{aligned}$$

Thus our solution is indeed correct.

24. We throw an object upward from the top of a 1200 ft tall building. The vertical position of the object, (measured in feet) t seconds after we threw it is

$$y = -16t^2 + 160t + 1200$$

a) Where is the object 3 seconds after we threw it?
Solution: We need to compute y when $t = 3$. This means that we substitute 3 into t and evaluate the algebraic expression.

$$\begin{aligned} y &= -16 \cdot 3^2 + 160 \cdot 3 + 1200 = -16 \cdot 9 + 160 \cdot 3 + 1200 \\ &= -144 + 480 + 1200 = 336 + 1200 = 1536 \end{aligned}$$

Thus the object is 1536 ft high after 3 seconds.

b) How long does it take for the object to hit the ground?
Solution: we need to solve the equation $t =?$ so that $y = 0$

$$\begin{aligned} y &= 0 \\ -16t^2 + 160t + 1200 &= 0 \qquad \text{factor out } -16 \\ -16\left(t^2 - 10t - 75\right) &= 0 \end{aligned}$$

We will factor $t^2 - 10t + 75$ by completing the square.

$$\begin{aligned} -16\left(t^2 - 10t - 75\right) &= 0 & (t-5)^2 = t^2 - 10t + 25 \quad \text{smuggle in 25} \\ -16\left(\underbrace{t^2 - 10t + 25} - 25 - 75\right) &= 0 & \\ -16\left((t-5)^2 - 100\right) &= 0 & \text{re-write 100 as } 10^2 \\ -16\left((t-5)^2 - 10^2\right) &= 0 & \text{factor via the difference of squares theorem} \\ -16\,(t - 5 + 10)\,(t - 5 - 10) &= 0 & \text{simplify} \\ -16\,(t+5)\,(t-15) &= 0 & \text{apply zero property} \end{aligned}$$

$$t = -5 \qquad \text{or} \qquad t = 15$$

Since the negative solution, $t = -5$ does not make sense in the context of the problem, it is ruled out. We check $t = 15$:

$$\begin{aligned} y &= -16 \cdot 15^2 + 160 \cdot 15 + 1200 \\ &= -16 \cdot 225 + 160 \cdot 15 + 1200 \\ &= -3600 + 2400 + 1200 \\ &= -1200 + 1200 = 0 \end{aligned}$$

Thus the answer is: 15 seconds.

