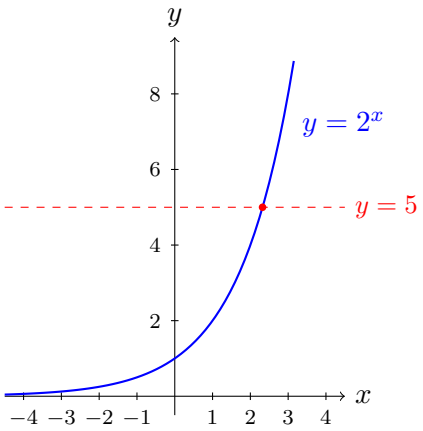
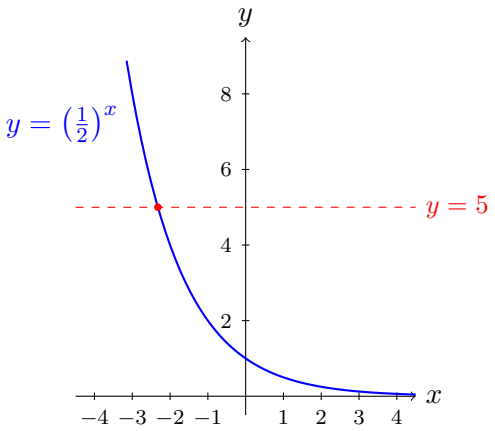
An **exponential equation** is an equation in which the unknown appears in the exponent, such as $2^x = 8$ or $3^{2x-1} = 10$. Some of these we can solve just by looking at them: $2^x = 8$ means that 2 raised to some power is 8, and since $2^3 = 8$, the solution is $x = 3$. But how do we know that there is no other solution? And what do we do with $3^{2x-1} = 10$, where 10 is not a nice power of 3?

Consider the function $f(x) = 2^x$. As x increases, 2^x also increases, so the graph of f is always going up (exponential growth). For a base between 0 and 1, such as $g(x) = \left(\frac{1}{2}\right)^x$, the graph is always going down (exponential decay). Either way, the graph can intersect any horizontal line at most once, so two different inputs can never give the same output. In other words, these functions are **one-to-one**.



exponential growth



exponential decay

This is exactly what we need: if two powers of the same base are equal, then their exponents must be equal. The same is true for any base b that is positive and different from 1.

Theorem. Suppose that $b > 0$ and $b \neq 1$. Then the function $f(x) = b^x$ is one-to-one. Therefore

$$\text{if} \quad b^u = b^v, \quad \text{then} \quad u = v.$$

Type 1: Both sides are powers of the same base

If we can write both sides of the equation as powers of the same base, then the theorem lets us drop the base and simply set the exponents equal to each other. We will need the exponent rules, and the facts

$$b^0 = 1, \qquad b^{-n} = \frac{1}{b^n}, \qquad \sqrt{b} = b^{\frac{1}{2}}, \qquad \sqrt[3]{b} = b^{\frac{1}{3}}, \qquad \text{and} \qquad (b^m)^n = b^{mn}.$$

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Example 1. Solve the equation $2^{3x-5} = 16$.

Solution: Since $16 = 2^4$, both sides are powers of 2.

$$\begin{aligned} 2^{3x-5} &= 16 && \text{write 16 as a power of 2} \\ 2^{3x-5} &= 2^4 && f(x) = 2^x \text{ is one-to-one} \\ 3x - 5 &= 4 && \text{add 5} \\ 3x &= 9 && \text{divide by 3} \\ x &= 3 \end{aligned}$$

Check: $2^{3\cdot 3-5} = 2^4 = 16$. The solution is $x = 3$.

Example 2. Solve the equation $3^{2x-1} = \dfrac{1}{27}$.

Solution: A fraction like $\dfrac{1}{27}$ is a power with a negative exponent: $\dfrac{1}{27} = \dfrac{1}{3^3} = 3^{-3}$.

$$\begin{aligned} 3^{2x-1} &= 3^{-3} && f(x) = 3^x \text{ is one-to-one} \\ 2x - 1 &= -3 && \text{add 1} \\ 2x &= -2 && \text{divide by 2} \\ x &= -1 \end{aligned}$$

The solution is $x = -1$.

Example 3. Solve each of the following equations. a) $2^{3x+1} = \sqrt{8}$ b) $2^{3x+1} = \dfrac{1}{\sqrt{8}}$

Solution: a) Roots are powers with fractional exponents: $\sqrt{8} = 8^{\frac{1}{2}} = \left(2^3\right)^{\frac{1}{2}} = 2^{\frac{3}{2}}$.

$$\begin{aligned} 2^{3x+1} &= 2^{\frac{3}{2}} && f(x) = 2^x \text{ is one-to-one} \\ 3x + 1 &= \frac{3}{2} && \text{subtract 1} \\ 3x &= \frac{1}{2} && \text{divide by 3} \\ x &= \frac{1}{6} \end{aligned}$$

b) Now the right-hand side is the reciprocal of the previous one, so its exponent is the opposite: $\dfrac{1}{\sqrt{8}} = \dfrac{1}{2^{\frac{3}{2}}} = 2^{-\frac{3}{2}}$.

$$\begin{aligned} 2^{3x+1} &= 2^{-\frac{3}{2}} && f(x) = 2^x \text{ is one-to-one} \\ 3x + 1 &= -\frac{3}{2} && \text{subtract 1} \\ 3x &= -\frac{5}{2} && \text{divide by 3} \\ x &= -\frac{5}{6} \end{aligned}$$

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Example 4. Solve the equation $9^{2x+1} = 27^{x+2}$.

Solution: Neither side is a power of the other, but both 9 and 27 are powers of 3: $9 = 3^2$ and $27 = 3^3$. We rewrite both sides as powers of 3, using $(b^m)^n = b^{mn}$.

$$\begin{aligned} \left(3^2\right)^{2x+1} &= \left(3^3\right)^{x+2} && \text{multiply the exponents} \\ 3^{4x+2} &= 3^{3x+6} && f(x) = 3^x \text{ is one-to-one} \\ 4x+2 &= 3x+6 && \text{subtract } 3x \text{ and } 2 \\ x &= 4 \end{aligned}$$

Check: $9^{2\cdot 4+1} = 9^9 = \left(3^2\right)^9 = 3^{18}$ and $27^{4+2} = 27^6 = \left(3^3\right)^6 = 3^{18}$. The solution is $x = 4$.

Example 5. Solve each of the following equations. a) $\left(\frac{1}{2}\right)^{x+1} = 8$ b) $0.25^x = 32$

Solution: Sometimes the common base is hidden. Here the bases $\frac{1}{2}$ and 0.25 are not powers of 8 or 32, but all four numbers are powers of 2.

a) $\frac{1}{2} = 2^{-1}$, so $\left(\frac{1}{2}\right)^{x+1} = \left(2^{-1}\right)^{x+1} = 2^{-x-1}$, and $8 = 2^3$.

$$\begin{aligned} 2^{-x-1} &= 2^3 && f(x) = 2^x \text{ is one-to-one} \\ -x-1 &= 3 && \text{add } 1 \\ -x &= 4 && \text{multiply by } -1 \\ x &= -4 \end{aligned}$$

b) $0.25 = \frac{1}{4} = 2^{-2}$, so $0.25^x = \left(2^{-2}\right)^x = 2^{-2x}$, and $32 = 2^5$.

$$\begin{aligned} 2^{-2x} &= 2^5 && f(x) = 2^x \text{ is one-to-one} \\ -2x &= 5 && \text{divide by } -2 \\ x &= -\frac{5}{2} \end{aligned}$$

Example 6. Solve the equation $2^{5x+1} = -4$.

Solution: A power of a positive base is always positive, so 2^{5x+1} can never be equal to a negative number. This equation has **no solution**. It is worth checking the sign of the other side before we start any computation.

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Example 7. Solve each of the following equations. a) $e^{3x+1} = \dfrac{1}{e^2}$ b) $e^{4x-1} = \sqrt{e}$

Solution: The number $e \approx 2.718$ is positive and different from 1, so $f(x) = e^x$ is also one-to-one, and everything works the same way with base e.

a) $\dfrac{1}{e^2} = e^{-2}$

$$\begin{aligned} e^{3x+1} &= e^{-2} && f(x) = e^x \text{ is one-to-one} \\ 3x+1 &= -2 && \text{subtract 1} \\ 3x &= -3 && \text{divide by 3} \\ x &= -1 \end{aligned}$$

b) $\sqrt{e} = e^{\frac{1}{2}}$

$$\begin{aligned} e^{4x-1} &= e^{\frac{1}{2}} && f(x) = e^x \text{ is one-to-one} \\ 4x-1 &= \frac{1}{2} && \text{add 1} \\ 4x &= \frac{3}{2} && \text{divide by 4} \\ x &= \frac{3}{8} \end{aligned}$$

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Type 2: One side is not a nice power

Consider now the equation $3^{2x-1} = 10$. The number 10 is not a power of 3 with a nice exponent: $3^2 = 9$ is a little less and $3^3 = 27$ is a lot more. Still, the graph of $f(x) = 3^x$ crosses the horizontal line $y = 10$ exactly once, so there *is* an exponent that makes 3 to that power equal to 10. This exponent has a name: it is the logarithm of 10 with base 3.

Definition. Suppose that $b > 0$, $b \neq 1$, and $a > 0$. The **logarithm** of a with base b, $\log_b a$, is the exponent we need to raise b to in order to get a:

$$\log_b a = c \qquad \text{means exactly the same as} \qquad b^c = a.$$

The logarithm with base 10 is called the **common logarithm**, written $\log a$, and the logarithm with base e is called the **natural logarithm**, written $\ln a$.

For example, $\log_2 8 = 3$ because $2^3 = 8$, and $\log_3 \frac{1}{27} = -3$ because $3^{-3} = \frac{1}{27}$. So for Type 2 equations, we walk away from the exponential statement and write the *equivalent logarithmic statement* instead. The exponent becomes "alone" on one side, and from there we solve a linear equation.

Example 8. Find the exact value of x if $3^{2x-1} = 10$.

Solution: The statement $3^{2x-1} = 10$ says that $2x - 1$ is the exponent we need to raise 3 to in order to get 10. That is exactly what $\log_3 10$ means.

$$\begin{aligned} 3^{2x-1} &= 10 && \text{write the equivalent logarithmic statement} \\ 2x - 1 &= \log_3 10 && \text{add 1} \\ 2x &= 1 + \log_3 10 && \text{divide by 2} \\ x &= \frac{1 + \log_3 10}{2} \end{aligned}$$

This is the exact value of the solution. (Since $3^2 = 9$ is a little less than 10, $\log_3 10$ is a little more than 2, so the solution is a little more than $\frac{3}{2}$.)

Example 9. Find the exact value of x if $e^{3x-1} = 10$.

Solution: The logarithm with base e is the natural logarithm, $\ln$.

$$\begin{aligned} e^{3x-1} &= 10 && \text{write the equivalent logarithmic statement} \\ 3x - 1 &= \ln 10 && \text{add 1} \\ 3x &= 1 + \ln 10 && \text{divide by 3} \\ x &= \frac{1 + \ln 10}{3} \end{aligned}$$

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Example 10. Find the exact value of x if $3 \cdot 5^{2x-1} + 4 = 10$.

Solution: Before we do anything with the exponent, we must isolate the power 5^{2x-1}: the 4 and the factor 3 are not part of it.

$$\begin{aligned} 3 \cdot 5^{2x-1} + 4 &= 10 && \text{subtract } 4 \\ 3 \cdot 5^{2x-1} &= 6 && \text{divide by } 3 \\ 5^{2x-1} &= 2 && \text{write the equivalent logarithmic statement} \\ 2x - 1 &= \log_5 2 && \text{add } 1 \\ 2x &= 1 + \log_5 2 && \text{divide by } 2 \\ x &= \frac{1 + \log_5 2}{2} \end{aligned}$$

Notice that after isolating the power, we could still have ended up with a Type 1 equation. For example, $3 \cdot 5^{2x-1} + 4 = 79$ leads to $5^{2x-1} = 25$, and then the exponents can be set equal: $2x - 1 = 2$.

Applications

Exponential equations come up whenever a quantity grows or decays by the same factor in equal time periods: populations, investments, radioactive materials.

Example 11. A bacteria population doubles every 3 hours. At the start of an experiment there are 100 bacteria. When will there be 800 bacteria?

Solution: After 3 hours there are $100 \cdot 2$ bacteria, after 6 hours $100 \cdot 2^2$, after 9 hours $100 \cdot 2^3$, and so on. After t hours, the population has doubled $\frac{t}{3}$ times, so the number of bacteria is $N(t) = 100 \cdot 2^{\frac{t}{3}}$.

$$\begin{aligned} 100 \cdot 2^{\frac{t}{3}} &= 800 && \text{divide by } 100 \\ 2^{\frac{t}{3}} &= 8 && \text{write } 8 \text{ as a power of } 2 \\ 2^{\frac{t}{3}} &= 2^3 && f(x) = 2^x \text{ is one-to-one} \\ \frac{t}{3} &= 3 && \text{multiply by } 3 \\ t &= 9 \end{aligned}$$

There will be 800 bacteria after 9 hours.

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Example 12. A radioactive substance has a half-life of 5 days: every 5 days, half of the remaining amount decays. We start with 200 milligrams. a) Find a formula for the amount left after t days. b) When will only 25 milligrams remain?

Solution: a) Let us see what happens in the first few half-lives.

t (days)	0	5	10	15	t
amount left (mg)	200	$200 \cdot \frac{1}{2}$	$200\left(\frac{1}{2}\right)^2$	$200\left(\frac{1}{2}\right)^3$	$200\left(\frac{1}{2}\right)^{\frac{t}{5}}$

After t days, $\frac{t}{5}$ half-lives have passed, so the amount has been halved $\frac{t}{5}$ times: $A(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{5}}$. We can check the formula: $A(0) = 200\left(\frac{1}{2}\right)^0 = 200$ and $A(10) = 200\left(\frac{1}{2}\right)^2 = 50$, as in the table.

b) Since $\frac{25}{200} = \frac{1}{8} = \left(\frac{1}{2}\right)^3$, this is a Type 1 equation.

$$\begin{aligned} 200\left(\frac{1}{2}\right)^{\frac{t}{5}} &= 25 && \text{divide by 200} \\ \left(\frac{1}{2}\right)^{\frac{t}{5}} &= \frac{1}{8} = \left(\frac{1}{2}\right)^3 && f(x) = \left(\frac{1}{2}\right)^x \text{ is one-to-one} \\ \frac{t}{5} &= 3 && \text{multiply by 5} \\ t &= 15 \end{aligned}$$

Only 25 milligrams will remain after 15 days, that is, after three half-lives.

Example 13. We invest \$5000 at 4% annual interest, compounded continuously. Then the value of the investment after t years is $A(t) = 5000e^{0.04t}$ dollars. When will the investment be worth \$8000?

Solution: $\frac{8000}{5000} = 1.6$ is not a nice power of e, so this is a Type 2 equation.

$$\begin{aligned} 5000e^{0.04t} &= 8000 && \text{divide by } 5000 \\ e^{0.04t} &= 1.6 && \text{write the equivalent logarithmic statement} \\ 0.04t &= \ln 1.6 && \text{divide by } 0.04 \\ t &= \frac{\ln 1.6}{0.04} = 25 \ln 1.6 \end{aligned}$$

With a calculator, $25 \ln 1.6 \approx 11.75$. The investment will be worth \$8000 after about 11.75 years.

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Summary. To solve an exponential equation,

1. isolate the power.
2. If the other side is zero or negative, there is no solution.
3. If both sides can be written as powers of the same base (Type 1), set the exponents equal.
4. Otherwise (Type 2), write the equivalent logarithmic statement.
5. Solve the resulting equation for x.



Practice Problems

Solve each of the following equations. Give the exact value of the solution.

1. $2^{x+3} = 32$
2. $5^{2x-1} = 125$
3. $3^{4x+1} = 81$
4. $2^{3x+2} = \dfrac{1}{16}$
5. $4^{x-1} = \dfrac{1}{64}$
6. $10^{2x-3} = 0.001$
7. $7^{5x-2} = 1$
8. $3^{2x+1} = \sqrt{3}$
9. $5^{x-2} = \dfrac{1}{\sqrt{5}}$
10. $2^{4x-1} = \sqrt[3]{4}$
11. $4^{2x-1} = \sqrt{8}$
12. $4^{x+1} = 8^{x-1}$
13. $25^{x+2} = 125^{x-1}$
14. $9^{x} = \left(\dfrac{1}{27}\right)^{x-5}$
15. $\left(\dfrac{1}{3}\right)^{x-2} = 27$
16. $0.5^{2x} = 16$
17. $\left(\dfrac{1}{4}\right)^{x} = 8$
18. $3^{x+2} = -9$
19. $e^{2x-5} = 1$
20. $e^{3x+2} = \dfrac{1}{e}$
21. $e^{4x-1} = \sqrt{e}$
22. $e^{x+1} = e^{2x-3}$
23. $2^{x-3} = 7$
24. $5^{2x+1} = 12$
25. $10^{3x} = 20$
26. $e^{4x-3} = 5$
27. $e^{2x} + 5 = 1$
28. $4 \cdot 3^{x-1} = 36$
29. $5 \cdot 2^{2x+1} + 3 = 23$
30. $2e^{3x} - 7 = 5$
31. $3 \cdot 2^{x+4} - 1 = 20$
32. A culture of 500 bacteria triples every 4 hours. a) Explain why the number of bacteria after t hours is $N(t) = 500 \cdot 3^{\frac{t}{4}}$. (Hint: compute $N(0)$, $N(4)$, and $N(8)$.) b) When will there be 13500 bacteria?
33. A radioactive substance has a half-life of 10 years: every 10 years, half of it decays. We start with 80 grams. a) Explain why the amount left after t years is $A(t) = 80\left(\dfrac{1}{2}\right)^{\frac{t}{10}}$ grams. (Hint: compute $A(0)$, $A(10)$, and $A(20)$.) b) When will only 5 grams remain?
34. We invest \$2000 at 5% annual interest, compounded continuously, so the value after t years is $A(t) = 2000e^{0.05t}$. When will the investment be worth \$3000? Give the exact value, and then round to two decimal places.
35. How long does it take for an investment to double at 6% annual interest, compounded continuously? (Use $A(t) = Pe^{0.06t}$.) Give the exact value, and then round to two decimal places.

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Enrichment

Example 14. Solve the equation $2^{x+1} + 2^x = 24$.

Solution: The left-hand side is a sum, so we cannot apply the theorem directly. But $2^{x+1} = 2 \cdot 2^x$, so both terms contain the factor 2^x.

$$2 \cdot 2^x + 2^x = 24 \implies 2^x (2+1) = 24 \implies 2^x = 8 = 2^3 \implies x = 3$$

Example 15. Solve the equation $4^x - 3 \cdot 2^x - 4 = 0$.

Solution: Since $4^x = \left(2^2\right)^x = (2^x)^2$, this is a quadratic equation in disguise. Let $u = 2^x$.

$$(2^x)^2 - 3 \cdot 2^x - 4 = 0 \implies u^2 - 3u - 4 = 0 \implies (u-4)(u+1) = 0$$

So $u = 4$ or $u = -1$. Now we go back to x. If $2^x = 4 = 2^2$, then $x = 2$. The other equation, $2^x = -1$, has no solution, because 2^x is always positive. So the only solution is $x = 2$.

Remark: An equation that becomes a quadratic equation $au^2 + bu + c = 0$ after a substitution $u =$ (some expression in x) is called **quadratic in form**. Other examples: $x^4 - 5x^2 + 4 = 0$ is quadratic in form with $u = x^2$, and $x - 3\sqrt{x} + 2 = 0$ is quadratic in form with $u = \sqrt{x}$.

Example 16. Find the exact value of x if $2^x = 3^{x-1}$.

Solution: The bases 2 and 3 are not powers of the same number. But we can use the exponent rules to collect everything with an x in the exponent on one side: $3^{x-1} = 3^x \cdot 3^{-1} = \dfrac{1}{3} \cdot 3^x$.

$$\begin{aligned}
2^x &= \frac{1}{3} \cdot 3^x && \text{divide by } 3^x \\
\frac{2^x}{3^x} &= \frac{1}{3} && \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x \\
\left(\frac{2}{3}\right)^x &= \frac{1}{3} && \text{write the equivalent logarithmic statement} \\
x &= \log_{\frac{2}{3}} \frac{1}{3}
\end{aligned}$$

The base $\dfrac{2}{3}$ is positive and different from 1, so this logarithm makes sense.

Solve each of the following equations.

36. $3^{x+2} - 3^x = 72$
37. $5^{x+1} + 5^{x-1} = 130$
38. $9^x - 10 \cdot 3^x + 9 = 0$
39. $4^x - 2^{x+1} - 8 = 0$
40. $e^{2x} - 5e^x + 6 = 0$
41. $4^x + 2^x - 2 = 0$
42. $5^x = 2^{x+1}$
43. $3^{x+1} = 4^x$

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Answers

1. $2^{x+3}=2^5,\ x+3=5,\ x=2$
2. $5^{2x-1}=5^3,\ 2x-1=3,\ x=2$
3. $3^{4x+1}=3^4,\ 4x+1=4,\ x=\dfrac{3}{4}$
4. $2^{3x+2}=2^{-4},\ 3x+2=-4,\ x=-2$
5. $4^{x-1}=4^{-3},\ x-1=-3,\ x=-2$
6. $10^{2x-3}=10^{-3},\ 2x-3=-3,\ x=0$
7. $7^{5x-2}=7^0,\ 5x-2=0,\ x=\dfrac{2}{5}$
8. $3^{2x+1}=3^{\frac{1}{2}},\ 2x+1=\dfrac{1}{2},\ x=-\dfrac{1}{4}$
9. $5^{x-2}=5^{-\frac{1}{2}},\ x-2=-\dfrac{1}{2},\ x=\dfrac{3}{2}$
10. $2^{4x-1}=2^{\frac{2}{3}},\ 4x-1=\dfrac{2}{3},\ x=\dfrac{5}{12}$
11. $2^{4x-2}=2^{\frac{3}{2}},\ 4x-2=\dfrac{3}{2},\ x=\dfrac{7}{8}$
12. $2^{2x+2}=2^{3x-3},\ 2x+2=3x-3,\ x=5$
13. $5^{2x+4}=5^{3x-3},\ 2x+4=3x-3,\ x=7$
14. $3^{2x}=3^{-3x+15},\ 2x=-3x+15,\ x=3$
15. $3^{-x+2}=3^3,\ -x+2=3,\ x=-1$
16. $2^{-2x}=2^4,\ -2x=4,\ x=-2$
17. $2^{-2x}=2^3,\ -2x=3,\ x=-\dfrac{3}{2}$
18. no solution, because $3^{x+2}>0$ for all x
19. $e^{2x-5}=e^0,\ 2x-5=0,\ x=\dfrac{5}{2}$
20. $e^{3x+2}=e^{-1},\ 3x+2=-1,\ x=-1$
21. $e^{4x-1}=e^{\frac{1}{2}},\ 4x-1=\dfrac{1}{2},\ x=\dfrac{3}{8}$
22. $x+1=2x-3,\ x=4$
23. $x-3=\log_2 7,\ x=3+\log_2 7$
24. $2x+1=\log_5 12,\ x=\dfrac{\log_5 12-1}{2}$
25. $3x=\log 20,\ x=\dfrac{\log 20}{3}$
26. $4x-3=\ln 5,\ x=\dfrac{3+\ln 5}{4}$
27. $e^{2x}=-4,$ no solution
28. $3^{x-1}=9=3^2,\ x-1=2,\ x=3$
29. $2^{2x+1}=4=2^2,\ 2x+1=2,\ x=\dfrac{1}{2}$
30. $e^{3x}=6,\ 3x=\ln 6,\ x=\dfrac{\ln 6}{3}$
31. $2^{x+4}=7,\ x+4=\log_2 7,\ x=\log_2 7-4$
32. a) $N(0)=500, N(4)=500\cdot 3, N(8)=500\cdot 3^2$: every 4 hours the exponent $\dfrac{t}{4}$ grows by 1, so the number triples. b) $500\cdot 3^{\frac{t}{4}}=13500,\ 3^{\frac{t}{4}}=27=3^3,$ $t=12$ hours
33. a) $A(0)=80, A(10)=40, A(20)=20$: every 10 years the exponent $\dfrac{t}{10}$ grows by 1, so the amount is halved. b) $80\left(\dfrac{1}{2}\right)^{\frac{t}{10}}=5,\ \left(\dfrac{1}{2}\right)^{\frac{t}{10}}=\dfrac{1}{16}=\left(\dfrac{1}{2}\right)^4,$ $t=40$ years
34. $e^{0.05t}=1.5,\ t=\dfrac{\ln 1.5}{0.05}=20\ln 1.5\approx 8.11$ years
35. $e^{0.06t}=2,\ t=\dfrac{\ln 2}{0.06}\approx 11.55$ years
36. $3^x(9-1)=72,\ 3^x=9,\ x=2$
37. $5^x\left(5+\dfrac{1}{5}\right)=130,\ 5^x=25,\ x=2$
38. $u=3^x$: $(u-1)(u-9)=0,\ x=0$ or $x=2$
39. $u=2^x$: $(u-4)(u+2)=0,\ 2^x=-2$ is impossible, $x=2$
40. $u=e^x$: $(u-2)(u-3)=0,\ x=\ln 2$ or $x=\ln 3$
41. $u=2^x$: $(u-1)(u+2)=0,\ x=0$
42. $5^x=2\cdot 2^x,\ \left(\dfrac{5}{2}\right)^x=2,\ x=\log_{\frac{5}{2}} 2$
43. $3\cdot 3^x=4^x,\ \left(\dfrac{4}{3}\right)^x=3,\ x=\log_{\frac{4}{3}} 3$

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