

Let $(a_n) = a_1, a_2, a_3, \dots$ be a geometric sequence. If we denote the first element a_1 by a and the common ratio by r , then the n th element a_n can be computed as

$$a_n = ar^{n-1}$$

and the sum of the first n th elements $s_n = a_1 + a_2 + a_3 + \dots + a_n$ can be computed as

$$\begin{aligned} s_n &= na & \text{if } r = 1 \\ s_n &= a \frac{1 - r^n}{1 - r} = a \frac{r^n - 1}{r - 1} & \text{if } r \neq 1 \end{aligned}$$

Sample Problems

- Let (a_n) be a geometric sequence with $a_1 = -3$ and $r = -2$. Compute each of the following.
a) a_9 b) a_{10} c) s_{15} d) s_{18} e) Find n if $a_n = -196\,608$
- A ship is 120 miles from land when its engine breaks. In the first hour, the ship covers 40 miles. After that, the engine loses 20% of its capacity in every hour. How long would it take for the ship to travel 75 miles with the broken engine?

Practice Problems

- Let (a_n) be a geometric sequence with $a_1 = 64\,000$ and $r = \frac{1}{2}$. Compute each of the following.
a) a_5 b) a_{14} c) s_{10} d) s_{15} e) Find n if $a_n = 125$.
- A ship is 900 miles from land when its engine breaks. In the first hour, the ship covers 60 miles. After that, the engine loses 5% of its capacity in every hour. How long would it take for the ship to travel 750 miles with the broken engine?

Sample Problems - Answers

1. a) -768 b) 1536 c) $-32\,769$ d) $262\,143$ e) 17
2. a little more than two hours

Practice Problems - Answers

1. a) 4000 b) $\frac{125}{16} = 7.8125$ c) $127\,875$ d) $127996.093\,75$ e) 10
2. a) $\frac{\ln \frac{3}{8}}{\ln 0.95} \approx 19.121\,98$ it would take a little more than nineteen hours

Sample Problems - Solutions

1. Let (a_n) be a geometric sequence with $a_1 = -3$ and $r = -2$. Compute each of the following.

a) $a_9 = a_1 r^8 = -3(-2)^8 = -768$

b) $a_{10} = a_1 r^9 = -3(-2)^9 = 1536$

c) Compute $s_{15} = a \frac{1 - r^{15}}{1 - r} = -3 \frac{1 - (-2)^{15}}{1 - (-2)} = -32\,769$

d) Compute $s_{18} = a \frac{1 - r^{18}}{1 - r} = -3 \frac{1 - (-2)^{18}}{1 - (-2)} = 262\,143$

e) Find n if $a_n = -196\,608$.

Solution: Solve $-3(-2)^x = -196\,608$ for x .

$$-3(-2)^x = -196\,608$$

$$(-2)^x = 65\,536$$

Because logarithms only work with positive numbers, we need to notice that x has to be even and in this case $(-2)^x = 2^x$. After this, we can solve this equation using logarithms.

$$2^x = 65\,536$$

$$\ln 2^x = \ln 65\,536$$

$$x \ln 2 = \ln 65\,536$$

$$x = \frac{\ln 65\,536}{\ln 2} = 16$$

Since $x = 16$, this means $a_n = a_1 r^{16}$ and so this is the 17th element.

2. A ship is 120 miles from land when its engine breaks. In the first hour, the ship covers 40 miles. After that, the engine loses 20% of its capacity in every hour. How long would it take for the ship to travel 75 miles with the broken engine?

Solution: We first find the formula for s_n , given the data above. Since the engine loses 20% in each hour, the engine's capacity decreases to its 80% in every hour. Thus, $a = 40$ and $r = 0.8$.

$$s_n = a \frac{1 - r^n}{1 - r} = 40 \frac{1 - 0.8^n}{1 - 0.8} = 40 \frac{1 - 0.8^n}{0.2} = \frac{40}{0.2} (1 - 0.8^n) = 200 (1 - 0.8^n)$$

$$75 = s_n$$

$$75 = 200 (1 - 0.8^n)$$

solve for n

$$\frac{75}{200} = 1 - 0.8^n$$

$$1 - \frac{75}{200} = 0.625$$

$$0.8^n = 0.625$$

$$\ln 0.8^n = \ln 0.625$$

$$n \ln 0.8 = \ln 0.625$$

$$n = \frac{\ln 0.625}{\ln 0.8} \approx 2.1063$$

It would take a little bit more than two hours to travel 75 miles.