# Sample Problems

Solve each of the following inequalities.

1.) $\dfrac{-2}{x-10} < 0$

2.) $\dfrac{x+7}{x-3} > 0$

3.) $\dfrac{x-1}{x+2} < 0$

4.) $\dfrac{p-5}{3-p} \leq 0$

5.) $\dfrac{2t+7}{t-4} \geq 3$

# Practice Problems

Solve each of the following inequalities.

1.) $\dfrac{a-1}{a} > 0$

2.) $\dfrac{3x+6}{2x-12} \leq 0$

3.) $\dfrac{-x+8}{x-2} \geq 5$

4.) $\dfrac{b+3}{5-2b} \leq 4$

5.) $\dfrac{3x-1}{x} \leq -1$

6.) $\dfrac{-2x+5}{x+6} > -2$

7.) $\dfrac{2}{p-1} \geq \dfrac{3}{4}$

8.) $\dfrac{2}{m+3} \leq 1$

# Sample Problems - Answers

1.) $x > 10$ - in interval notation: $(10, \infty)$

2.) $x < -7$ or $x > 3$ - in interval notation: $(-\infty, -7) \cup (3, \infty)$

3.) $-2 < x < 1$ - in interval notation: $(-2, 1)$

4.) $p < 3$ or $p \geq 5$ - in interval notation: $(-\infty, 3) \cup [5, \infty)$

5.) $4 < t \leq 19$ - in interval notation: $(4, 19]$

# Practice Problems - Answers

1.) $a < 0$ or $a > 1$ - in interval notation: $(-\infty, 0) \cup (1, \infty)$

2.) $-2 \leq x < 6$ - in interval notation: $[-2, 6)$

3.) $2 < x \leq 3$ - in interval notation: $(2, 3]$

4.) $b \leq \frac{17}{9}$ or $b > \frac{5}{2}$ - in interval notation: $\left(-\infty, \frac{17}{9}\right] \cup \left(\frac{5}{2}, \infty\right)$

5.) $0 < x \leq \frac{1}{4}$ - in interval notation: $\left(0, \frac{1}{4}\right]$

6.) $x > -6$ - in interval notation: $(-6, \infty)$

7.) $1 < p \leq \frac{11}{3}$ - in interval notation: $\left(1, \frac{11}{3}\right]$

8.) $m < -3$ or $m \geq -1$ - in interval notation: $(-\infty, -3) \cup [-1, \infty)$

# Sample Problems - Solutions

Solve each of the following inequalities.

1.) $\dfrac{-2}{x-10} < 0$

Solution: The numerator, $-2$ is always negative. The denominator, $x-10$ is negative when $x < 10$ and positive when $x > 10$. We summerize these in the table shown below.

| | $x < 10$ | $x > 10$ |
|---|---|---|
| the numerator $-2$ | $-$ | $-$ |
| the denominator $x-10$ | $-$ | $+$ |
| the quotient $\dfrac{-2}{x-10}$ | $+$ | $-$ |

We can now see that the quotient is negative when $x > 10$. The same solution can be written in interval notation as $(10, \infty)$.

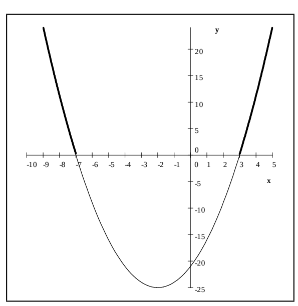
2.) $\dfrac{x+7}{x-3} > 0$

Solution: Method 1. The numerator, $x+7$ is negative when $x < -7$ and positive when $x > -7$. The denominator, $x-3$ is negative when $x < 3$ and positive when $x > 3$. We summerize these in the table shown below.

| | $x < -7$ | $-7 < x < 3$ | $x > 3$ |
|---|---|---|---|
| the numerator $x+7$ | $-$ | $+$ | $+$ |
| the denominator $x-3$ | $-$ | $-$ | $+$ |
| the quotient $\dfrac{x+7}{x-3}$ | $+$ | $-$ | $+$ |

We can now see that the quotient is positive when $x < -7$ or when $x > 3$. The same solution can be written in interval notation as $(-\infty, -7) \cup (3, \infty)$.

Method 2. (Note that this method only works if one side of the inequality is zero.) This method is based on the fact that the inequalities $\dfrac{x+7}{x-3} > 0$ and $(x+7)(x-3) > 0$ have the same solution. We simply solve the quadratic inequality by graphing the parabola $y = (x+7)(x-3)$ and observe when the graph is above the $x-$axis.




The graph of $y = (x+7)(x-3)$

This is clearly when $x < -7$ or when $x > 3$. The same solution can be written in interval notation as $(-\infty, -7) \cup (3, \infty)$.

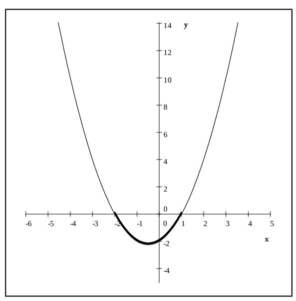
3.) $\dfrac{x-1}{x+2} < 0$

Solution: Method 1. The numerator, $x-1$ is negative when $x < 1$ and positive when $x > 1$. The denominator, $x+2$ is negative when $x < -2$ and positive when $x > -2$. We summerize these in the table shown below.

| | $x < -2$ | $-2 < x < 1$ | $x > 1$ |
|---|---|---|---|
| the numerator $x-1$ | $-$ | $-$ | $+$ |
| the denominator $x+2$ | $-$ | $+$ | $+$ |
| the quotient $\dfrac{x-1}{x+2}$ | $+$ | $-$ | $+$ |

We can now see that the quotient is negative when $-2 < x < 1$. The same solution can be written in interval notation as $(-2, 1)$.

Method 2. (Note that this method only works if one side of the inequality is zero.) This method is based on the fact that the inequalities $\dfrac{x-1}{x+2} < 0$ and $(x+2)(x-1) < 0$ have the same solution. We simply solve the quadratic inequality by graphing the parabola $y = (x+2)(x-1)$ and observe when the graph is below the $x-$axis.




The graph of $y = (x+2)(x-1)$

This is clearly when $-2 < x < 1$. The same solution can be written in interval notation as $(-2, 1)$.

4.) $\dfrac{p-5}{3-p} \le 0$

Solution: Method 1. Let us first re-write the quotient $\dfrac{p-5}{3-p}$ as $\dfrac{p-5}{-(p-3)}$.

We will solve the inequality in two parts:

Part 1. $\dfrac{p-5}{-(p-3)} < 0$ and Part 2. $\dfrac{p-5}{-(p-3)} = 0$.

Part 1. The numerator, $p-5$ is negative when $p < 5$ and positive when $p > 5$. The denominator, $-(p-3)$ is positive when $p < 3$ and negative when $p > 3$. We summerize these in the table shown below.

| | $p < 3$ | $3 < p < 5$ | $p > 5$ |
|---|---|---|---|
| the numerator $p-5$ | $-$ | $-$ | $+$ |
| the denominator $-(p-3)$ | $+$ | $-$ | $-$ |
| the quotient $\dfrac{p-5}{-(p-3)}$ | $-$ | $+$ | $-$ |

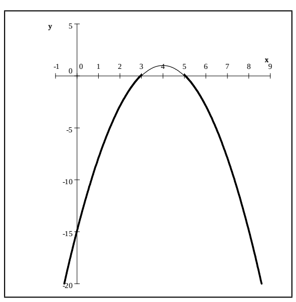
We can now see that the quotient is negative when $p < 3$ or when $p > 5$. The same solution can be written in interval notation as $(-\infty, 3) \cup (5, \infty)$.

Part 2. $\dfrac{p-5}{-(p-3)} = 0$

We have to find all values of $p$ that make the fraction $\dfrac{p-5}{-(p-3)}$ zero. There are only two candidates: the value of $p$ that makes the numerator zero, is $p = 5$ and the value of $p$ that makes the denominator zero, is $p = 3$. We have to consider only these two values. We find that $p = 5$ makes the fraction zero and is therefore a solution, and that $p = 3$ makes the fraction undefined and is NOT a solution. In short, a fraction $\dfrac{A}{B}$ can be zero if and only if the numerator $A$ is zero.

The solution of the inequality $\dfrac{p-5}{3-p} \leq 0$ is therefore $p < 3$ or $p \geq 5$. The same solution can be written in interval notation as $(-\infty, 3) \cup [5, \infty)$.

Method 2. Let us first re-write the quotient $\dfrac{p-5}{3-p}$ as $\dfrac{p-5}{-(p-3)}$.
We will solve the inequality in two parts:
Part 1. $\dfrac{p-5}{-(p-3)} < 0$ and Part 2 $\dfrac{p-5}{-(p-3)} = 0$.
Part 1. (Note that this method only works if one side of the inequality is zero.) This method is based on the fact that the inequalities $\dfrac{p-5}{-(p-3)} < 0$ and $-(p-3)(p-5) < 0$ have the same solution. We simply solve the quadratic inequality by graphing the parabola $y = -(x-5)(x-3)$ and observe when the graph is below the $x-$axis.



The graph of $y = -(x-3)(x-5)$

This is clearly when $x < 3$ or when $x > 5$. The same solution can be written in interval notation as $(-\infty, 3) \cup (5, \infty)$.
Part 2. $\dfrac{p-5}{-(p-3)} = 0$

We have to find all values of $p$ that make the fraction $\dfrac{p-5}{-(p-3)}$ zero. There are only two candidates: the value of $p$ that makes the numerator zero, is $p = 5$ and the value of $p$ that makes the denominator zero, is $p = 3$. We have to consider only these two values. We find that $p = 5$ makes the fraction zero and is therefore a solution, and that $p = 3$ makes the fraction undefined and is NOT a solution. In short, a fraction $\dfrac{A}{B}$ can be zero if and only if the numerator $A$ is zero.

The solution of the inequality $\dfrac{p-5}{-(p-3)} \leq 0$ is therefore $p < 3$ or $p \geq 5$. The same solution can be written in interval notation as $(-\infty, 3) \cup [5, \infty)$.

5.) $\dfrac{2t+7}{t-4} \geq 3$

Solution: So far, the methods of solving rational inequalities have been based on determining the sign (positive or negative) of a quotient by counting the negative signs. This ultimately requires that one side of the inequality is zero. If the inequality is NOT like that, our first task is to transform it to such a form. This requires a little bit of algebra.

$$\begin{aligned}
\frac{2t+7}{t-4} &\geq 3 && \text{subtract 3} \\
\frac{2t+7}{t-4} - 3 &\geq 0 && \text{bring to common denominator} \\
\frac{2t+7}{t-4} - \frac{3(t-4)}{t-4} &\geq 0 \\
\frac{2t+7-3(t-4)}{t-4} &\geq 0 \\
\frac{2t+7-3t+12}{t-4} &\geq 0 \\
\frac{-t+19}{t-4} &\geq 0 \\
\frac{-(t-19)}{t-4} &\geq 0
\end{aligned}$$

The inequality $\dfrac{-(t-19)}{t-4} \geq 0$ is one that we can easily solve now. Using either of the methods presented in the previous problems, we easily obtain that $\dfrac{-(t-19)}{t-4} > 0$ when $4 < t < 19$ and $\dfrac{-(t-19)}{t-4} = 0$ when $t = 19$. Thus the solution of the inequality is $4 < t \leq 19$, or, in interval notation, $(4, 19]$.

