

Sample Problems

1. $\sqrt{3x-2} = x$

6. $2\sqrt{x+4} = 1 + \sqrt{2x+9}$

11. $\sqrt{3x+1} - \sqrt{x-4} = 3$

2. $\sqrt[3]{3x-6} = 3$

7. $5\sqrt{x+1} = 3\sqrt{x} + 17$

12. $\sqrt{x+10} + 10 = x$

3. $10 + \sqrt{4x-7} = 7$

8. $\sqrt{2x+5} + 5 = x$

13. $\sqrt[3]{x^3+208} = x+4$

4. $5 + \sqrt{x+15} = x$

9. $\sqrt{2x+5} - \sqrt{x-1} = \sqrt{x+2}$

14. $\sqrt{x-1} + \sqrt{x-4} = \sqrt{4x-11}$

5. $2\sqrt{x-1} = x-4$

10. $\sqrt[3]{x^3+26} = x+2$

15. $\sqrt{4x+6} = \sqrt{x+1} - \sqrt{x+5}$

Practice Problems

1. $\sqrt{3x-5} = 4$

9. $\sqrt{w} + \sqrt{w+3} = 3$

17. $\sqrt{3k-5} - \sqrt{3k} = -1$

2. $2\sqrt{a-1} + 7 = 1$

10. $\sqrt{2x+1} + \sqrt{5-x} = 4$

18. $\sqrt{1-x} + 1 = x+12$

3. $3\sqrt{7x+1} + 2 = 20$

11. $2\sqrt{x} - \sqrt{x-3} = \sqrt{x+7}$

19. $2 = \sqrt{x^2+1} - x$

4. $\sqrt{x+3} = x-9$

12. $2\sqrt{y+4} + \sqrt{y-5} = \sqrt{9y+7}$

20. $\sqrt{3x+1} - 4 = 8 - 2\sqrt{3x+1}$

5. $2\sqrt{x+5} = x-3$

13. $\sqrt{b-2} + b = 8$

21. $\sqrt{x} = \sqrt{10+3\sqrt{x}}$

6. $\sqrt{2p+4} - \sqrt{p+3} = 1$

14. $\sqrt{3x+1} - \sqrt{x+4} = 1$

22. $\sqrt[3]{x^3+7} = x+1$

7. $\sqrt[3]{x^3+16} = x+4$

15. $\sqrt[3]{x-8} - \sqrt[3]{4x+1} = 0$

23. $\sqrt{x-6} = \sqrt{x+2} - 4$

8. $\sqrt{18+x} = x-2$

16. $\sqrt{5m-9} = \sqrt{5m} - 3$

24. $\sqrt{6x+7} - \sqrt{3x+3} = 1$

Sample Problems – Answers

- 1.) 1, 2 2.) 11 3.) no real solution 4.) 10 5.) 10 6.) 0 7.) 64 8.) 10
9.) 2 10.) -3, 1 11.) 5, 8 12.) 15 13.) -6, 2 14.) 5 15.) no real solution

Practice Problems – Answers

- 1.) 7 2.) no real solution 3.) 5 4.) 13 5.) 11 6.) 6 7.) -2 8.) 7 9.) 1
10.) $4, \frac{20}{9}$ 11.) $\frac{25}{8}$ 12.) 21 13.) 6 14.) 5 15.) -3 16.) $\frac{9}{5}$ 17.) 3
18.) -8 19.) $-\frac{3}{4}$ 20.) 5 21.) 25 22.) -2, 1 23.) no real solution 24.) $\frac{1}{3}, -1$

Sample Problems – Solutions

1. $\sqrt{3x-2} = x$ 1,2

Solution:

$$\begin{aligned}
 \sqrt{3x-2} &= x && \text{square} \\
 3x-2 &= x^2 && \text{reduce one side to zero} \\
 0 &= x^2 - 3x + 2 && \text{factor} \\
 0 &= (x-2)(x-1) \implies x_1 = 2 \text{ and } x_2 = 1
 \end{aligned}$$

We check: if $x = 2$, then

$$\begin{aligned}
 \text{LHS} &= \sqrt{3(2)-2} = \sqrt{4} = 2 \\
 \text{RHS} &= 2
 \end{aligned}$$

and if $x = 1$, then

$$\begin{aligned}
 \text{LHS} &= \sqrt{3(1)-2} = \sqrt{1} = 1 \\
 \text{RHS} &= 1
 \end{aligned}$$

Thus the solution set is: $\{1, 2\}$

2. $\sqrt[3]{3x-6} = 3$ 11

Solution:

$$\begin{aligned}
 \sqrt[3]{3x-6} &= 3 && \text{raise both sides to the third power} \\
 3x-6 &= 27 && \text{add 6} \\
 3x &= 33 && \text{divide by 3} \\
 x &= 11
 \end{aligned}$$

We check: if $x = 11$, then

$$\text{LHS} = \sqrt[3]{3(11)-6} = \sqrt[3]{27} = 3 = \text{RHS}$$

3. $10 + \sqrt{4x-7} = 7$ no real solution

Solution:

$$\begin{aligned}
 10 + \sqrt{4x-7} &= 7 && \text{subtract 10} \\
 \sqrt{4x-7} &= -3
 \end{aligned}$$

Since the square root of no real number is negative, there is no real solution.

4. $5 + \sqrt{x+15} = x$ 10

Solution:

$$\begin{aligned}
 5 + \sqrt{x+15} &= x && \text{subtract 5} \\
 \sqrt{x+15} &= x-5 && \text{square both sides} \\
 x+15 &= (x-5)^2 && \text{FOIL right hand side} \\
 x+15 &= x^2 - 10x + 25 && \text{reduce one side to zero} \\
 0 &= x^2 - 11x + 10 && \text{factor} \\
 0 &= (x-1)(x-10) \implies x_1 = 1 \text{ and } x_2 = 10
 \end{aligned}$$

We check: If $x = 1$, then

$$\begin{aligned}\text{LHS} &= 5 + \sqrt{1 + 15} = 5 + \sqrt{16} = 5 + 4 = 9 \\ \text{RHS} &= 1 \\ \text{RHS} &\neq \text{LHS}\end{aligned}$$

Thus $x = 1$ is NOT a solution.

If $x = 10$, then

$$\begin{aligned}\text{LHS} &= 5 + \sqrt{10 + 15} = 5 + \sqrt{25} = 5 + 5 = 10 \\ \text{RHS} &= 10 \\ \text{RHS} &= \text{LHS}\end{aligned}$$

Thus $x = 10$ is the only solution.

5. $2\sqrt{x-1} = x - 4$ 10

Solution:

$$\begin{aligned}2\sqrt{x-1} &= x - 4 && \text{square both sides} \\ 4(x-1) &= (x-4)^2 && \text{FOIL, distribute} \\ 4x - 4 &= x^2 - 8x + 16 && \text{reduce one side to zero} \\ 0 &= x^2 - 12x + 20 && \text{factor} \\ 0 &= (x-2)(x-10) && \implies x_1 = 2 \text{ and } x_2 = 10\end{aligned}$$

We check: If $x = 2$, then

$$\begin{aligned}\text{LHS} &= 2\sqrt{2-1} = 2\sqrt{1} = 2(1) = 2 \\ \text{RHS} &= 2 - 4 = -2 \\ \text{RHS} &\neq \text{LHS}\end{aligned}$$

Thus $x = 2$ is NOT a solution.

If $x = 10$, then

$$\begin{aligned}\text{LHS} &= 2\sqrt{10-1} = 2\sqrt{9} = 2(3) = 6 \text{ and } \text{RHS} = 10 - 4 = 6 \\ \text{RHS} &= \text{LHS}\end{aligned}$$

Thus $x = 10$ is the only solution.

6. $2\sqrt{x+4} = 1 + \sqrt{2x+9}$ 0

Solution:

$$\begin{aligned}2\sqrt{x+4} &= 1 + \sqrt{2x+9} && \text{square both sides} \\ (2\sqrt{x+4})^2 &= (1 + \sqrt{2x+9})^2 \\ 2^2(\sqrt{x+4})^2 &= (1 + \sqrt{2x+9})(1 + \sqrt{2x+9}) \\ 4(x+4) &= 1 + \sqrt{2x+9} + \sqrt{2x+9} + 2x + 9 && \text{combine like terms} \\ 4x + 16 &= 2x + 10 + 2\sqrt{2x+9} && \text{subtract } 2x \\ 2x + 16 &= 10 + 2\sqrt{2x+9} && \text{subtract } 10 \\ 2x + 6 &= 2\sqrt{2x+9} && \text{factor out } 2\end{aligned}$$

$$\begin{array}{ll}
2(x+3) &= 2\sqrt{2x+9} && \text{divide by 2} \\
x+3 &= \sqrt{2x+9} && \text{square both sides} \\
(x+3)^2 &= 2x+9 \\
x^2+6x+9 &= 2x+9 && \text{reduce one side to zero} \\
x^2+4x &= 0 && \text{factor} \\
x(x+4) &= 0 \implies x_1 = 0 \quad \text{and} \quad x_2 = -4
\end{array}$$

We check: If $x = 0$, then

$$\text{LHS} = 2\sqrt{0+4} = 2\sqrt{4} = 2 \cdot 2 = 4 \quad \text{and} \quad \text{RHS} = 1 + \sqrt{2(0)+9} = 1 + 3 = 4$$

Thus $x = 0$ is indeed a solution.

If $x = -4$, then

$$\begin{array}{ll}
\text{LHS} &= 2\sqrt{(-4)+4} = 2\sqrt{0} = 2(0) = 0 \\
\text{RHS} &= 1 + \sqrt{2(-4)+9} = 1 + \sqrt{-8+9} = 1 + \sqrt{1} = 1 + 1 = 2 \\
\text{RHS} &\neq \text{LHS}
\end{array}$$

Thus $x = -4$ is NOT a solution. The only solution is $x = 0$.

7. $5\sqrt{x} + 1 = 3\sqrt{x} + 17$ 64

Solution:

$$\begin{array}{ll}
5\sqrt{x} + 1 &= 3\sqrt{x} + 17 && \text{subtract } 3\sqrt{x} \\
2\sqrt{x} + 1 &= 17 && \text{subtract 1} \\
2\sqrt{x} &= 16 && \text{divide by 2} \\
\sqrt{x} &= 8 && \text{square both sides} \\
x &= 64
\end{array}$$

We check: If $x = 64$, then

$$\text{LHS} = 5\sqrt{64} + 1 = 5(8) + 1 = 41 \quad \text{and} \quad \text{RHS} = 3\sqrt{64} + 17 = 3(8) + 17 = 24 + 17 = 41$$

Thus $x = 64$ is indeed a solution.

8. $\sqrt{2x+5} + 5 = x$ 10

Solution:

$$\begin{array}{ll}
\sqrt{2x+5} + 5 &= x && \text{subtract 5} \\
\sqrt{2x+5} &= x - 5 && \text{square} \\
2x + 5 &= x^2 - 10x + 25 && \text{reduce one side to zero} \\
0 &= x^2 - 12x + 20 && \text{factor} \\
0 &= (x-2)(x-10) \implies x_1 = 2 \quad \text{and} \quad x_2 = 10
\end{array}$$

We check: If $x = 2$, then

$$\begin{array}{ll}
\text{LHS} &= \sqrt{2(2)+5} + 5 = \sqrt{4+5} + 5 = \sqrt{9} + 5 = 3 + 5 = 8 \quad \text{and} \quad \text{RHS} = 2 \\
\text{RHS} &\neq \text{LHS}
\end{array}$$

Thus $x = 2$ is NOT a solution.

If $x = 10$, then

$$\begin{array}{ll}
\text{LHS} &= \sqrt{2(10)+5} + 5 = \sqrt{20+5} + 5 = \sqrt{25} + 5 = 5 + 5 = 10 \\
\text{RHS} &= 10 \\
\text{RHS} &= \text{LHS}
\end{array}$$

Thus $x = 10$ is the only solution.

9. $\sqrt{2x+5} - \sqrt{x-1} = \sqrt{x+2}$ **2**

Solution:

$$\begin{aligned}
 \sqrt{2x+5} - \sqrt{x-1} &= \sqrt{x+2} && \text{add } \sqrt{x-1} \\
 \sqrt{2x+5} &= \sqrt{x+2} + \sqrt{x-1} && \text{square} \\
 (\sqrt{2x+5})^2 &= (\sqrt{x+2} + \sqrt{x-1})^2 \\
 2x+5 &= (\sqrt{x+2} + \sqrt{x-1})(\sqrt{x+2} + \sqrt{x-1}) \\
 2x+5 &= \underbrace{\sqrt{x+2}\sqrt{x+2}}_{\text{F}} + \underbrace{\sqrt{x+2}\sqrt{x-1}}_{\text{O}} + \underbrace{\sqrt{x-1}\sqrt{x+2}}_{\text{I}} + \underbrace{\sqrt{x-1}\sqrt{x-1}}_{\text{L}} \\
 2x+5 &= x+2 + 2\sqrt{x-1}\sqrt{x+2} + x-1 \\
 2x+5 &= 2x+1 + 2\sqrt{x-1}\sqrt{x+2} && \text{subtract } 2x \\
 5 &= 1 + 2\sqrt{(x-1)(x+2)} && \text{subtract } 1 \\
 4 &= 2\sqrt{(x-1)(x+2)} && \text{divide by } 2 \\
 2 &= \sqrt{(x-1)(x+2)} && \text{square} \\
 4 &= (x-1)(x+2) && \text{FOIL} \\
 4 &= x^2 + x - 2 && \text{reduce one side to zero} \\
 0 &= x^2 + x - 6 && \text{factor} \\
 0 &= (x+3)(x-2) \implies x_1 = -3 \text{ and } x_2 = 2
 \end{aligned}$$

We check: If $x = -3$, then

$$\text{LHS} = \sqrt{2(-3)+5} - \sqrt{(-3)-1} = \sqrt{-1} - \sqrt{-4} = \text{undefined}$$

Since the left hand side is undefined, $x = -3$ is NOT a solution.

If $x = 2$, then

$$\begin{aligned}
 \text{LHS} &= \sqrt{2(2)+5} - \sqrt{2-1} = \sqrt{9} - \sqrt{1} = 3 - 1 = 2 \\
 \text{RHS} &= \sqrt{2+2} = \sqrt{4} = 2 \\
 \text{RHS} &= \text{LHS}
 \end{aligned}$$

Thus $x = 2$ is the only solution.

10. $\sqrt[3]{x^3+26} = x+2$ **-3, 1**

Solution:

$$\begin{aligned}
 (\sqrt[3]{x^3+26})^3 &= (x+2)^3 \\
 x^3+26 &= (x+2)^3 \\
 x^3+26 &= x^3+6x^2+12x+8 \\
 0 &= x^3+6x^2+12x+8-x^3-26 \\
 0 &= 6x^2+12x-18 \\
 0 &= 6(x^2+2x-3) \\
 0 &= 6(x+3)(x-1) \implies x_1 = -3 \text{ and } x_2 = 1
 \end{aligned}$$

We check both answers: if $x = -3$, then

$$\begin{aligned}
 \text{LHS} &= \sqrt[3]{(-3)^3+26} = \sqrt[3]{-27+26} = \sqrt[3]{-1} = -1 \\
 \text{RHS} &= (-3)+2 = -1
 \end{aligned}$$

thus -3 does work. If $x = 1$, then

$$\begin{aligned}\text{LHS} &= \sqrt[3]{1^3 + 26} = \sqrt[3]{1 + 26} = \sqrt[3]{27} = 3 \\ \text{RHS} &= 1 + 2 = 3\end{aligned}$$

The solution set is $\{-3, 1\}$.

11. $\sqrt{3x+1} - \sqrt{x-4} = 3$ 5, 8

Solution:

$$\begin{aligned}\sqrt{3x+1} - \sqrt{x-4} &= 3 && \text{add } \sqrt{x-4} \text{ to both sides} \\ \sqrt{3x+1} &= 3 + \sqrt{x-4} && \text{square both sides} \\ 3x+1 &= (3 + \sqrt{x-4})^2 \\ 3x+1 &= (3 + \sqrt{x-4})(3 + \sqrt{x-4}) \\ 3x+1 &= 9 + 3\sqrt{x-4} + 3\sqrt{x-4} + x - 4 \\ 3x+1 &= x + 5 + 6\sqrt{x-4} && \text{subtract } x \\ 2x+1 &= 5 + 6\sqrt{x-4} && \text{subtract } 5 \\ 2x-4 &= 6\sqrt{x-4} \\ 2(x-2) &= 6\sqrt{x-4} && \text{divide by } 2 \\ x-2 &= 3\sqrt{x-4} && \text{square both sides} \\ (x-2)^2 &= 9(x-4) && \text{FOIL, distribute} \\ x^2 - 4x + 4 &= 9x - 36 && \text{reduce one side to zero} \\ x^2 - 13x + 40 &= 0 && \text{factor} \\ (x-5)(x-8) &= 0 \implies x_1 = 5 \text{ and } x_2 = 8\end{aligned}$$

We check: if $x = 5$, then

$$\text{LHS} = \sqrt{3(5)+1} - \sqrt{5-4} = \sqrt{16} - \sqrt{1} = 4 - 1 = 3 = \text{RHS}$$

If $x = 8$, then

$$\text{LHS} = \sqrt{3(8)+1} - \sqrt{8-4} = \sqrt{25} - \sqrt{4} = 5 - 2 = 3 = \text{RHS}$$

The solution set is $\{5, 8\}$.

12. $\sqrt{x+10} + 10 = x$ 15

Solution:

$$\begin{aligned}\sqrt{x+10} + 10 &= x && \text{subtract } 10 \\ \sqrt{x+10} &= x - 10 && \text{square} \\ x + 10 &= (x - 10)^2 \\ x + 10 &= x^2 - 20x + 100 && \text{reduce one side to zero} \\ 0 &= x^2 - 21x + 90 && \text{factor} \\ 0 &= (x - 6)(x - 15) \implies x_1 = 6 \text{ and } x_2 = 15\end{aligned}$$

We check: if $x = 6$, then

$$\begin{aligned}\text{LHS} &= \sqrt{6+10} + 10 = \sqrt{16} + 10 = 4 + 10 = 14 \\ \text{RHS} &= 6 \\ \text{LHS} &\neq \text{RHS}\end{aligned}$$

If $x = 15$, then

$$\text{LHS} = \sqrt{15 + 10} + 10 = \sqrt{25} + 10 = 5 + 10 = 15 = \text{RHS}$$

since $x = 6$ doesn't work, the only solution is 15. In set notation: the solution set is $\{15\}$.

13. $\sqrt[3]{x^3 + 208} = x + 4$ **-6, 2**

Solution:

$$\begin{aligned} \sqrt[3]{x^3 + 208} &= x + 4 && \text{raise both sides to the third power} \\ \left(\sqrt[3]{x^3 + 208}\right)^3 &= (x + 4)^3 \\ x^3 + 208 &= x^3 + 12x^2 + 48x + 64 && \text{subtract } x^3 \\ 208 &= 12x^2 + 48x + 64 && \text{reduce one side to zero} \\ 0 &= 12x^2 + 48x - 144 && \text{factor out 12} \\ 0 &= 12(x^2 + 4x - 12) && \text{factor} \\ 0 &= 12(x + 6)(x - 2) \implies x_1 = -6 \text{ and } x = 2 \end{aligned}$$

Solution: -6 and 2. We check. They both work.

14. $\sqrt{x-1} + \sqrt{x-4} = \sqrt{4x-11}$ **5**

Solution:

$$\begin{aligned} \sqrt{x-1} + \sqrt{x-4} &= \sqrt{4x-11} && \text{square} \\ (\sqrt{x-1} + \sqrt{x-4})^2 &= (\sqrt{4x-11})^2 \\ (\sqrt{x-1} + \sqrt{x-4})(\sqrt{x-1} + \sqrt{x-4}) &= 4x - 11 && \text{FOIL} \\ \underbrace{\sqrt{x-1}\sqrt{x-1}}_{\text{F}} + \underbrace{\sqrt{x-1}\sqrt{x-4}}_{\text{O}} + \underbrace{\sqrt{x-4}\sqrt{x-1}}_{\text{I}} + \underbrace{\sqrt{x-4}\sqrt{x-4}}_{\text{L}} &= 4x - 11 \\ x - 1 + 2\sqrt{x-1}\sqrt{x-4} + x - 4 &= 4x - 11 && \text{combine like terms} \\ 2x - 5 + 2\sqrt{x-1}\sqrt{x-4} &= 4x - 11 && \text{subtract } 2x \\ -5 + 2\sqrt{x-1}\sqrt{x-4} &= 2x - 11 && \text{add 5} \\ 2\sqrt{x-1}\sqrt{x-4} &= 2x - 6 \\ 2\sqrt{x-1}\sqrt{x-4} &= 2(x - 3) && \text{divide by 2} \\ \sqrt{(x-1)(x-4)} &= x - 3 && \text{square} \\ (x-1)(x-4) &= (x-3)^2 && \text{FOIL} \\ x^2 - 5x + 4 &= x^2 - 6x + 9 && \text{subtract } x^2 \\ -5x + 4 &= -6x + 9 && \text{add } 6x \\ x + 4 &= 9 && \text{subtract 4} \\ x &= 5 \end{aligned}$$

We check:

$$\begin{aligned} \text{LHS} &= \sqrt{5-1} + \sqrt{5-4} = \sqrt{4} + \sqrt{1} = 2 + 1 = 3 \\ \text{RHS} &= \sqrt{4(5)-11} = \sqrt{20-11} = \sqrt{9} = 3 \end{aligned}$$

And so the solution set is $\{5\}$.

15. $\sqrt{4x+6} = \sqrt{x+1} - \sqrt{x+5}$ **no real solution**

Solution:

$$\begin{aligned}
 \sqrt{4x+6} &= \sqrt{x+1} - \sqrt{x+5} && \text{square} \\
 (\sqrt{4x+6})^2 &= (\sqrt{x+1} - \sqrt{x+5})^2 \\
 4x+6 &= \underbrace{\sqrt{x+1}\sqrt{x+1}}_{\text{F}} - \underbrace{\sqrt{x+1}\sqrt{x+5}}_{\text{O}} - \underbrace{\sqrt{x+5}\sqrt{x+1}}_{\text{I}} + \underbrace{\sqrt{x+5}\sqrt{x+5}}_{\text{L}} \\
 4x+6 &= x+1 - 2\sqrt{(x+1)(x+5)} + x+5 \\
 4x+6 &= 2x+6 - 2\sqrt{(x+1)(x+5)} && \text{subtract } 2x \\
 2x+6 &= 6 - 2\sqrt{(x+1)(x+5)} && \text{subtract } 6 \\
 2x &= -2\sqrt{(x+1)(x+5)} && \text{divide by } 2 \\
 x &= -\sqrt{(x+1)(x+5)} && \text{square} \\
 x^2 &= \left(-\sqrt{(x+1)(x+5)}\right)^2 \\
 x^2 &= (x+1)(x+5) && \text{FOIL right hand side} \\
 x^2 &= x^2 + 6x + 5 && \text{subtract } x^2 \\
 0 &= 6x + 5 && \text{subtract } 5 \\
 -5 &= 6x && \text{divide by } 6 \\
 -\frac{5}{6} &= x
 \end{aligned}$$

We check: if $x = -\frac{5}{6}$, then

$$\begin{aligned}
 \text{LHS} &= \sqrt{4\left(-\frac{5}{6}\right) + 6} = \sqrt{-\frac{10}{3} + 6} = \sqrt{-\frac{10}{3} + \frac{18}{3}} = \sqrt{\frac{8}{3}} \\
 \text{RHS} &= \sqrt{-\frac{5}{6} + 1} - \sqrt{-\frac{5}{6} + 5} = \sqrt{\frac{1}{6}} - \sqrt{\left(-\frac{5}{6}\right) + \frac{30}{6}} = \sqrt{\frac{1}{6}} - \sqrt{\frac{25}{6}} = \frac{\sqrt{1}}{\sqrt{6}} - \frac{\sqrt{25}}{\sqrt{6}} \\
 &= \frac{1}{\sqrt{6}} - \frac{5}{\sqrt{6}} = \frac{1-5}{\sqrt{6}} = -\frac{4}{\sqrt{6}}
 \end{aligned}$$

Since the left hand side is positive, and the right hand side is negative, these two numbers can not be equal. This equation has no real solution.