Sample Problems

Solve each of the following equations.

1. $3^{2x-1} = \dfrac{1}{27}$
2. $2^{\frac{1}{3}x+1} = 32$
3. $3^{2x-1} = 10$
4. $e^{3x-1} = 10$
5. $2^{5x+1} = -1$
6. $3^{2x-10} = 3^{x-2}$
7. $3^{3x-1} = 9^{x-1}$
8. $e^{-\ln x} = 5$
9. $\log_2(3x-1) = 5$
10. $\dfrac{1}{2}\ln(2x+5) - 3 = 4$
11. $\dfrac{\ln(2x+5) - 3}{2} = 4$
12. $\dfrac{3}{4}\log_2(5x+1) - 1 = 5$

Practice Problems

Solve each of the following equations.

1. $2^{3x+1} = 128$
2. $2^{2x-1} = \dfrac{1}{8}$
3. $2^{5x-1} = 4^{x-2}$
4. $2^{3x-2} = 50$
5. $2^{3x-1} = -4$
6. $e^{5x-1} = 28$
7. $4^{2x-1} = \sqrt{8}$
8. $4^{\log_2 x} = 7$
9. $3^{5x-1} = 81$
10. $5^{2x-3} = \dfrac{1}{5}$
11. $5^{x-2} = -5$
12. $5^{2x+8} = 22$
13. $5^{3x-1} = 25^{x-5}$
14. $e^{2x-1} = \sqrt{e}$
15. $\log_2(2x-1) = -2$
16. $\dfrac{\log_3(2x+1) - 3}{2} = -1$
17. $\dfrac{\log_3(2x+1)}{2} - 3 = -1$

Answers - Sample Problems

1.) -1 2.) 12 3.) $\frac{1}{2}(1+\log_3 10)$ 4.) $\frac{1}{3}(1+\ln 10)$ 5.) no solution 6.) 8 7.) -1 8.) $\frac{1}{5}$

9.) 11 10.) $\frac{1}{2}\left(e^{14}-5\right)$ 11.) $\frac{1}{2}\left(e^{11}-5\right)$ 12.) 51

Answers - Practice Problems

1.) 2 2.) $-\frac{2}{3}$ 3.) -1 4.) $\frac{1}{3}(2+\log_2 50)$ 5.) no solution 6.) $\frac{1}{5}(1+\ln 28)$ 7.) $\frac{2}{3}$

8.) $\sqrt{7}$ 9.) 1 10.) 1 11.) no solution 12.) $\frac{1}{2}(-8+\log_5 22)$ 13.) -9 14.) $\frac{3}{4}$ 15.) $\frac{5}{8}$

16.) 1 17.) 40

Sample Problems - Solutions

1. $3^{2x-1} = \dfrac{1}{27}$

Solution: In this problem we will use the fact that $f(x) = 3^x$ is a one-to-one function. That means that is two 3−powers are equal, then the exponents must be the same.

$$\begin{aligned} 3^{2x-1} &= 3^{-3} && f(x) = 3^x \text{ is one-to-one} \\ 2x-1 &= -3 && \text{solve for } x \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

We check: if $x = -1$, then $3^{2(-1)-1} = 3^{-3} = \dfrac{1}{27}$ so our solution is correct.

2. $2^{\frac{1}{3}x+1} = 32$

Solution:

$$\begin{aligned} 2^{\frac{1}{3}x+1} &= 32 && f(x) = 2^x \text{ is one-to-one} \\ 2^{\frac{1}{3}x+1} &= 2^5 \\ \frac{1}{3}x + 1 &= 5 && \text{solve for } x \\ \frac{1}{3}x &= 4 \\ x &= 12 \end{aligned}$$

We check: if $x = 12$, then $2^{\frac{1}{3}(12)+1} = 2^{4+1} = 2^5 = 32$ so our solution is correct.

3. $3^{2x-1} = 10$

Solution: we re-write the exponential statement as a logarithmic statement and solve for x.

$$\begin{aligned} 3^{2x-1} &= 10 \\ \log_3 10 &= 2x - 1 \\ 1 + \log_3 10 &= 2x \\ \frac{1 + \log_3 10}{2} &= x \end{aligned}$$

We check: if $x = \dfrac{1}{2}(1 + \log_3 10)$, then $3^{2\frac{1}{2}(1+\log_3 10)-1} = 3^{\log_3 10} = 10$ so our solution is correct.

4. $e^{3x-1} = 10$

Solution: we re-write the exponential statement as a logarithmic statement and solve for x.

$$\begin{aligned} e^{3x-1} &= 10 \\ \ln 10 &= 3x - 1 \\ 1 + \ln 10 &= 3x \\ \frac{1 + \ln 10}{3} &= x \end{aligned}$$

We check: if $x = \dfrac{1}{3}(1 + \ln 10)$, then $e^{3\frac{1}{3}(1+\ln 10)-1} = e^{\ln 10} = 10$ so our solution is correct.

5. $2^{5x+1} = -1$

   Solution: This equation has no solution because no $2-$power can be negative.

6. $3^{2x-10} = 3^{x-2}$

   Solution: In this problem we will use the fact that $f(x) = 3^x$ is a one-to-one function. That means that is two $3-$powers are equal, then the exponents must be the same.

$$\begin{aligned} 3^{2x-10} &= 3^{x-2} && f(x) = 3^x \text{ is one-to-one} \\ 2x - 10 &= x - 2 && \text{solve for } x \\ x - 10 &= -2 \\ x &= 8 \end{aligned}$$

   We check: if $x = 8$, then

$$\text{LHS} = 3^{2(8)-10} = 3^{16-10} = 3^6 \quad \text{and} \quad \text{RHS} = 3^{8-2} = 3^6$$

   so our solution is correct.

7. $3^{3x-1} = 9^{x-1}$

   Solution: In this problem we will use the fact that $f(x) = 3^x$ is a one-to-one function. That means that is two $3-$powers are equal, then the exponents must be the same.

$$\begin{aligned} 3^{3x-1} &= 9^{x-1} \\ 3^{3x-1} &= \left(3^2\right)^{x-1} \\ 3^{3x-1} &= 3^{2(x-1)} && f(x) = 3^x \text{ is one-to-one} \\ 3x - 1 &= 2(x-1) && \text{solve for } x \\ 3x - 1 &= 2x - 2 \\ x - 1 &= -2 \\ x &= -1 \end{aligned}$$

   We check: if $x = -1$, then

$$\text{LHS} = 3^{3(-1)-1} = 3^{-4} = \frac{1}{81} \quad \text{and} \quad \text{RHS} = 9^{-1-1} = 9^{-2} = \frac{1}{81}$$

   so our solution is correct.

8. $e^{-\ln x} = 5$

   Solution: We will use the fact that $e^{\ln x} = x$. So the left-hand side can be re-written as $e^{-\ln x} = e^{\ln x(-1)} = \left(e^{\ln x}\right)^{-1} = x^{-1} = \dfrac{1}{x}$

$$\begin{aligned} \frac{1}{x} &= 5 \\ 1 &= 5x \\ \frac{1}{5} &= x \end{aligned}$$

   We check: if $x = \dfrac{1}{5}$, then

$$\text{LHS} = e^{-\ln\left(\frac{1}{5}\right)} = \left(e^{\ln\left(\frac{1}{5}\right)}\right)^{-1} = \left(\frac{1}{5}\right)^{-1} = 5 = \text{RHS}$$

   so our solution is correct.

9. $\log_2(3x-1)=5$

Solution: There are several methods for solving this problem. We will start by re-stating the logarithmic statement as an exponential one and then solve for x. Recall that a logarithmic statement can always be re-stated as an exponential one: $\log_A B = C$ implies that $A^C = B$.

$$\begin{aligned} \log_2(3x-1) &= 5 && \text{re-state as exponential} \\ 2^5 &= 3x-1 && \text{solve for } x \\ 32 &= 3x-1 && \text{add 1} \\ 33 &= 3x && \text{divide by 3} \\ 11 &= x \end{aligned}$$

We check: if $x=11$, then

$$\text{LHS} = \log_2(3\cdot 11-1) = \log_2(33-1) = \log_2 32 = 5 = \text{RHS}$$

so our solution is correct.

10. $\dfrac{1}{2}\ln(2x+5)-3=4$

Solution: Before we can re-write a logarithmic statement as an exponential one, we must isolate the logarithmic expression. So we will add 3 and multiply by 2 first to isolate the logarithmic expression. Then we re-state it as an exponential statement and solve for x.

$$\begin{aligned} \frac{1}{2}\ln(2x+5)-3 &= 4 && \text{add 3} \\ \frac{1}{2}\ln(2x+5) &= 7 && \text{multiply by 2} \\ \ln(2x+5) &= 14 && \text{re-state as exponential} \\ e^{14} &= 2x+5 && \text{subtract 5} \\ e^{14}-5 &= 2x && \text{divide by 2} \\ \frac{e^{14}-5}{2} &= x \end{aligned}$$

$\dfrac{e^{14}-5}{2}$ is the exact value of the solution. The approximate value can be found using the calculator: $\dfrac{e^{14}-5}{2} \approx 601299.642\,082$. We will use the exact value for checking

$$\text{LHS} = \frac{1}{2}\ln\left(2\left(\frac{e^{14}-5}{2}\right)+5\right)-3 = \frac{1}{2}\ln\left(e^{14}-5+5\right)-3 = \frac{1}{2}\ln\left(e^{14}\right)-3 = \frac{1}{2}\cdot 14-3 = 7-3 = 4 = \text{RHS}$$

and so our solution is correct.

11. $\dfrac{\ln(2x+5)-3}{2}=4$

Solution: This problem is different from the previous one in what is needed to isolate the logarithmic expresssion. This time we need to multiply by 2 first and then add 3.

$$\begin{aligned} \frac{\ln(2x+5)-3}{2} &= 4 && \text{multiply by 2} \\ \ln(2x+5)-3 &= 8 && \text{add 3} \\ \ln(2x+5) &= 11 && \text{re-state as exponential} \\ e^{11} &= 2x+5 && \text{subtract 5} \\ e^{11}-5 &= 2x && \text{divide by 2} \\ \frac{e^{11}-5}{2} &= x \end{aligned}$$

We check:

$$\text{LHS} = \frac{\ln\left(2\left(\dfrac{e^{11}-5}{2}\right)+5\right)-3}{2} = \frac{\ln\left(e^{11}-5+5\right)-3}{2} = \frac{\ln\left(e^{11}\right)-3}{2} = \frac{11-3}{2} = \frac{8}{2} = 4 = \text{RHS}$$

12. $\dfrac{3}{4}\log_2(5x+1)-1=5$

Solution: We will first isolate the logarithmic expression, then re-write it as an exponential statement, and finally, solve for x.

$$\begin{aligned}
\frac{3}{4}\log_2(5x+1)-1 &= 5 && \text{add 1}\\
\frac{3}{4}\log_2(5x+1) &= 6 && \text{divide by } \frac{3}{4}\\
\log_2(5x+1) &= 8 && \text{re-state as exponential}\\
2^8 &= 5x+1 &&\\
256 &= 5x+1 && \text{subtract 1}\\
255 &= 5x && \text{divide by 5}\\
51 &= x &&
\end{aligned}$$

We check:

$$\text{LHS} = \frac{3}{4}\log_2(5\cdot 51+1)-1 = \frac{3}{4}\log_2(256)-1 = \frac{3}{4}\log_2 256-1 = \frac{3}{4}\log_2\left(2^8\right)-1 = \frac{3}{4}\cdot 8-1 = 6-1 = 5 = \text{RHS}$$

