Sample Problems

Solve each of the following inequalities.

1.) $\dfrac{x+3}{x-4} \geq 2$

2.) $\dfrac{x+7}{x-3} > 0$

3.) $\dfrac{5t-1}{t-2} \leq 3$

Practice Problems

Solve each of the following inequalities.

1.) $\dfrac{a-1}{a} > 0$

2.) $\dfrac{3x+6}{2x-12} \leq 0$

3.) $\dfrac{-x+8}{x-2} \geq 5$

4.) $\dfrac{b+3}{5-2b} \leq 4$

5.) $\dfrac{3x-1}{x} \leq -1$

6.) $\dfrac{-2x+5}{x+6} > -2$

7.) $\dfrac{2}{p-1} \geq \dfrac{3}{4}$

8.) $\dfrac{2}{m+3} \leq 1$

Sample Problems - Answers

1.) $4 < x \leq 11$ - in interval notation: $(4, 11]$

2.) $x < -7$ or $x > 3$ - in interval notation: $(-\infty, -7) \cup (3, \infty)$

3.) $-\dfrac{5}{2} \leq x < 2$ - in interval notation: $\left[-\dfrac{5}{2}, 2\right)$

Practice Problems - Answers

1.) $a < 0$ or $a > 1$ - in interval notation: $(-\infty, 0) \cup (1, \infty)$

2.) $-2 \leq x < 6$ - in interval notation: $[-2, 6)$

3.) $2 < x \leq 3$ - in interval notation: $(2, 3]$

4.) $b \leq \dfrac{17}{9}$ or $b > \dfrac{5}{2}$ - in interval notation: $\left(-\infty, \dfrac{17}{9}\right] \cup \left(\dfrac{5}{2}, \infty\right)$

5.) $0 < x \leq \dfrac{1}{4}$ - in interval notation: $\left(0, \dfrac{1}{4}\right]$

6.) $x > -6$ - in interval notation: $(-6, \infty)$

7.) $1 < p \leq \dfrac{11}{3}$ - in interval notation: $\left(1, \dfrac{11}{3}\right]$

8.) $m < -3$ or $m \geq -1$ - in interval notation: $(-\infty, -3) \cup [-1, \infty)$

Sample Problems - Solutions

Solve each of the following inequalities.

1.) $\dfrac{x+3}{x-4} \geq 2$

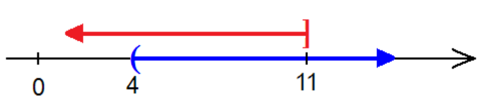
Solution: We would like to multiply both sides by $x-4$ but we have to be careful about how to do that. The problem is that $x-4$ may be positive or negative and so when multiplying by it, we do not know whether to keep the inequality sign or reverse it. The answer is that we will investigat both possibilities. First of all, let us notice that $x-4=0$ that is, $x=4$ will not be a solution because it makes the left-hand side undefined. In other words, $x=4$ is not even in the domain of the expression. There are two cases left for us to consider: when the expression $x-4$ is positive and when it is negative.

Case 1. When $x-4$ is positive, i.e. when $x-4>0$.

We first solve this inequality for x and obtain $x>4$. This means that in this case, we are looking for solutions only **among the real numbers that are greater than 4**. When we multiply both sides by $x-4$, we keep the inequality sign the same.

$$\begin{aligned} \frac{x+3}{x-4} &\geq 2 && \text{multiply by } x-4 \\ x+3 &\geq 2(x-4) && \\ x+3 &\geq 2x-8 && \text{subtract } x \\ 3 &\geq x-8 && \text{add } 8 \\ 11 &\geq x && \end{aligned}$$

We found that among the numbers greatar than 4, the solutions will be the numbers that are less than or equal to 11.



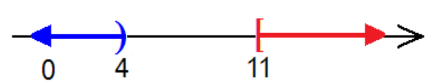
Those are the numbers $4<x\leq 11$ or, in interval notation, $(4,11]$.

Case 2. When $x-4$ is negative, i.e. when $x-4<0$.

We first solve this inequality for x and obtain $x<4$. This means that in this case, we are looking for solutions only **among the real numbers less than 4**. When we multiply both sides by $x-4$, we must reverse the inequality sign.

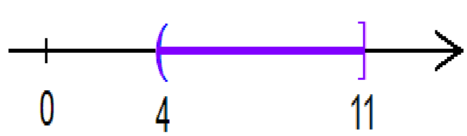
$$\begin{aligned} \frac{x+3}{x-4} &\geq 2 && \text{multiply by } x-4 \\ x+3 &\leq 2(x-4) && \\ x+3 &\leq 2x-8 && \text{subtract } x \\ 3 &\leq x-8 && \text{add } 8 \\ 11 &\leq x && \end{aligned}$$

We found that among the numbers less than 4, the solutions will be the numbers that are greater than or equal to 11.



There are no such numbers and so this case yields no solution.

That means that the final answer is $4<x\leq 11$ or, in interval notation, $(4,11]$.



2.) $\dfrac{x+7}{x-3} > 0$

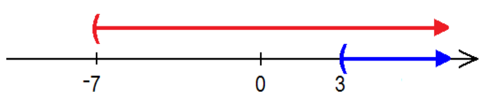
Solution: We would like to multiply both sides by $x-3$ but we have to be careful about how to do that. The quantity $x-3$ may be positive or negative and so when multiplying by it, we have to do that carefully. First of all, $x=3$ will not be a solution because it makes the left-hand side undefined. There are two cases left for us to consider: when the expression $x-3$ is positive and when it is negative.

Case 1. When $x-3$ is positive.

We first solve the inequality $x-3>0$ for x and obtain $x>3$.
This means that in this case, we are looking for solutions only **among the real numbers that are greater than 3**. When we multiply both sides by $x-3$, we keep the inequality sign the same.

$$\begin{aligned} \frac{x+7}{x-3} &> 0 && \text{multiply by } x-3 \\ x+7 &> 0\,(x-3) \\ x+7 &> 0 && \text{subtract } 7 \\ x &> -7 \end{aligned}$$

We found that among the numbers greater than 3, the solutions will be the numbers that are greater than -7.



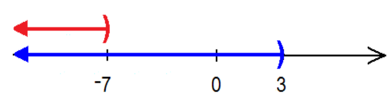
Those are the numbers $x>3$ or in interval notation, $(3,\infty)$.

Case 2. When $x-3$ is negative.

We first solve the inequality $x-3<0$ for x and obtain $x<3$.
This means that in this case, we are looking for solutions only **among the real numbers less than 3**. When we multiply both sides by $x-3$, we must reverse the inequality sign.

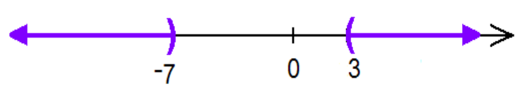
$$\begin{aligned} \frac{x+7}{x-3} &> 0 && \text{multiply by } x-3 \\ x+7 &< 0\,(x-3) \\ x+7 &< 0 && \text{subtract } 7 \\ x &< -7 \end{aligned}$$

We found that among the numbers less than 3, the solutions will be the numbers that are less than -7.



Those are the numbers $x<-7$, or, in interval notation, $(-\infty,-7)$.

The solution set is the union of these two sets, $x<-7$ or $x>-3$,or, in interval notation, $(-\infty,-7)\cup(3,\infty)$.



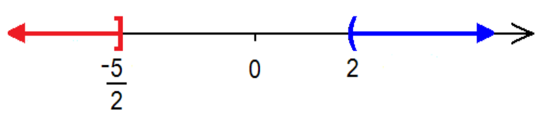
3.) $\dfrac{5t-1}{t-2} \leq 3$

Case 1. When $t-2>0$.

We first solve this inequality for t and obtain $t>2$. This means that in this case, we are looking for solutions only **among the real numbers that are greater than 2**. When we multiply both sides by $t-2$, we keep the inequality sign the same.

$$\begin{aligned} \frac{5t-1}{t-2} &\leq 3 && \text{multiply by } t-2 \\ 5t-1 &\leq 3(t-2) \\ 5t-1 &\leq 3t-6 && \text{subtract } 3t \\ 2t-1 &\leq -6 && \text{add } 1 \\ 2t &\leq -5 && \text{divide by } 2 \\ t &\leq -\frac{5}{2} \end{aligned}$$

We found that among the numbers greater than 2, the solutions will be the numbers that are less than or equal to $-\dfrac{5}{2}$.



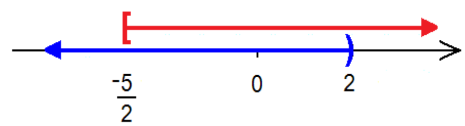
There are no such numbers and so this case yields no solution.

Case 2. When $t-2<0$.

We first solve this inequality for t and obtain $t<2$. This means that in this case, we are looking for solutions only **among the real numbers less than 2**. When we multiply both sides by $t-2$, we must reverse the inequality sign.

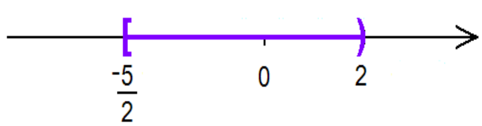
$$\begin{aligned} \frac{5t-1}{t-2} &\leq 3 && \text{multiply by } t-2 \\ 5t-1 &\geq 3(t-2) \\ 5t-1 &\geq 3t-6 && \text{subtract } 3t \\ 2t-1 &\geq -6 && \text{add } 1 \\ 2t &\geq -5 && \text{divide by } 2 \\ t &\geq -\frac{5}{2} \end{aligned}$$

We found that among the numbers less than 2, the solutions will be the numbers that are greater than or equal to $-\dfrac{5}{2}$.



These numbers are $-\dfrac{5}{2} \leq x < 2$ or in interval notation: $\left[-\dfrac{5}{2}, 2\right)$.

That means that the final answer is $-\dfrac{5}{2} \leq x < 2$ or in interval notation: $\left[-\dfrac{5}{2}, 2\right)$.



For more documents like this, visit our page at https://teaching.martahidegkuti.com and click on Lecture Notes. E-mail questions or comments to mhidegkuti@ccc.edu.

