Sample Problems

1. Divide $x^4 + x^3 - 5x^2 + 26x - 21$ by $x^2 + 3x - 4$

Practice Problems

Perform each of the following divisions with remainder.

1. $\left(3x^5 - 8x^4 + 11x^2 + 2x - 7\right) \div (x - 2)$
2. $\left(3x^5 - 8x^4 + 11x^2 + 2x - 7\right) \div \left(x^2 + x\right)$
3. $\left(3x^5 - 8x^4 + 11x^2 + 2x - 7\right) \div \left(x^3 - 3x\right)$
4. $\left(3x^5 - 8x^4 + 11x^2 + 2x - 7\right) \div \left(x^2 + 1\right)$
5. $\left(5x^5 - 24x^3 - 7x^2 - 5x + 35\right) \div (x - 1)$
6. $\left(5x^5 - 24x^3 - 7x^2 - 5x + 35\right) \div \left(x^3 + 2x - 1\right)$
7. $\left(5x^5 - 24x^3 - 7x^2 - 5x + 35\right) \div \left(x^2 - 5\right)$
8. $\left(5x^5 - 24x^3 - 7x^2 - 5x + 35\right) \div (x + 3)$
9. $\left(x^6 - 1\right) \div (x - 1)$
10. $\left(x^6 - 1\right) \div (x + 2)$
11. $\left(x^6 - 1\right) \div \left(x^2 - 4\right)$
12. $\left(x^2 - 7x + 16\right) \div (x - 3)$
13. $\left(x^3 - 5x^2 + 8x - 1\right) \div (x - 3)$
14. $\left(x^4 - 5x^3 + 4x^2 + x + 8\right) \div (x - 1)$
15. $\left(3x^4 - x^3 + 5x - 4\right) \div (x + 1)$
16. $\left(x^5 - 2\right) \div (x - 2)$
17. $\left(5x^3 - 2x^2 + 5x - 7\right) \div \left(x^2 - 3\right)$
18. $\left(x^4 - 3x^3 + x^2 - 8x - 2\right) \div (x + 2)$
19. $\left(x^4 - 4x^3 + 10x^2 - 7\right) \div (x + 1)$
20. $\left(x^4 - 4x^3 + 10x^2 - 7\right) \div (x - 2)$

Answers - Sample Problems

1. $x^2 - 2x + 5$ R $3x - 1$

Answers - Practice Problems

1. $3x^4 - 2x^3 - 4x^2 + 3x + 8$ R 9
2. $3x^3 - 11x^2 + 11x$ R $2x - 7$
3. $3x^2 - 8x + 9$ R $-13x^2 + 29x - 7$
4. $3x^3 - 8x^2 - 3x + 19$ R $5x - 26$
5. $5x^4 + 5x^3 - 19x^2 - 26x - 31$ R 4
6. $5x^2 - 34$ R $-2x^2 + 63x + 1$
7. $5x^3 + x - 7$
8. $5x^4 - 15x^3 + 21x^2 - 70x + 205$ R -580
9. $x^5 + x^4 + x^3 + x^2 + x + 1$
10. $x^5 - 2x^4 + 4x^3 - 8x^2 + 16x - 32$ R 63
11. $x^4 + 4x^2 + 16$ R 63
12. $x - 4$ R 4
13. $x^2 - 2x + 2$ R 5
14. $x^3 - 4x^2 + 1$ R 9
15. $3x^3 - 4x^2 + 4x + 1$ R -5
16. $x^4 + 2x^3 + 4x^2 + 8x + 16$ R 30
17. $5x - 2$ R $20x - 13$
18. $x^3 - 5x^2 + 11x - 30$ R $\dfrac{58}{x+2}$
19. $x^3 - 5x^2 + 15x - 15$ R 8
20. $x^3 - 2x^2 + 6x + 12$ R 17

Sample Problems - Solutions

1. Divide $x^4 + x^3 - 5x^2 + 26x - 21$ by $x^2 + 3x - 4$

   Step 1: Divide the first term by the first term: $\dfrac{x^4}{x^2} = x^2$

   x^2 is the first term of the quotient; we write it above the line.

   We multiply x^2 and what we are dividing by: $x^2\left(x^2 + 3x - 4\right) = x^4 + 3x^3 - 4x^2$

   We take the opposite of the result: $-\left(x^4 + 3x^3 - 4x^2\right) = -x^4 - 3x^3 + 4x^2$

   We add that to the original polynomial shown above. This is now the new polynomial to divide, and the process begins again.

$$\begin{array}{rrrrrrrrrr}
 & x^2 & & & & & & & & \\ \cline{2-10}
x^2 + 3x - 4 \;) & x^4 & + & x^3 & - & 5x^2 & + & 26x & - & 21 \\
 & -x^4 & - & 3x^3 & + & 4x^2 & & & & \\ \cline{2-10}
 & & - & 2x^3 & - & x^2 & + & 26x & - & 21
\end{array}$$

   Step 2: We now need to divide $-2x^3 - x^2 + 26x - 21$ by $x^2 + 3x - 4$. Divide the first term by the first term: $\dfrac{-2x^3}{x^2} = -2x$

   $-2x$ is the second term of the quotient; we write it above the line.

   We multiply $-2x$ and what we are dividing by: $-2x\left(x^2 + 3x - 4\right) = -2x^3 - 6x^2 + 8x$

   We take the opposite of the result: $-1\left(-2x^3 - 6x^2 + 8x\right) = 2x^3 + 6x^2 - 8x$

   We add that to the polynomial to be divided. This sum is now the new polynomial to divide, and the process begins again.

$$\begin{array}{rrrrrrrrrr}
 & x^2 & - & 2x & & & & & & \\ \cline{2-10}
x^2 + 3x - 4 \;) & x^4 & + & x^3 & - & 5x^2 & + & 26x & - & 21 \\
 & -x^4 & - & 3x^3 & + & 4x^2 & & & & \\ \cline{2-10}
 & & - & 2x^3 & - & x^2 & + & 26x & - & 21 \\
 & & & 2x^3 & + & 6x^2 & - & 8x & & \\ \cline{4-10}
 & & & & & 5x^2 & + & 18x & - & 21
\end{array}$$

   Step 3: Divide the first term by the first term: $\dfrac{5x^2}{x^2} = 5$

   5 is the third term of the quotient; we write it above the line.

   We multiply 5 and what we are dividing by: $5\left(x^2 + 3x - 4\right) = 5x^2 + 15x - 20$

   We take the opposite of the result: $-1\left(5x^2 + 15x - 20\right) = -5x^2 - 15x + 20$

   We add that to the polynomial to be divided.

$$\begin{array}{rrrrrrrrrr}
 & x^2 & - & 2x & + & 5 & & & & \\ \cline{2-10}
x^2 + 3x - 4 \;) & x^4 & + & x^3 & - & 5x^2 & + & 26x & - & 21 \\
 & -x^4 & - & 3x^3 & + & 4x^2 & & & & \\ \cline{2-10}
 & & - & 2x^3 & - & x^2 & + & 26x & - & 21 \\
 & & & 2x^3 & + & 6x^2 & - & 8x & & \\ \cline{4-10}
 & & & & & 5x^2 & + & 18x & - & 21 \\
 & & & & - & 5x^2 & - & 15x & + & 20 \\ \cline{5-10}
 & & & & & & & 3x & - & 1
\end{array}$$

For the next step, we would have to divide $3x-1$ by x^2+3x-4. The division of the first terms $\dfrac{3x}{x^2}=\dfrac{3}{x}$ is not allowed: this is where we must stop. The quotient is x^2-2x+5 and the remainder is $3x-1$. The result can be presented in two different ways, very much like the division $38 \div 5$ can be represented in two ways:

$$\frac{38}{5}=7\frac{3}{5} \quad \text{or} \quad 38 \div 5 = 7 \text{ R } 3$$

our result is

$$\frac{x^4+x^3-5x^2+26x-21}{x^2+3x-4}=\boxed{x^2-2x+5+\frac{3x-1}{x^2+3x-4}}$$

or

$$\left(x^4+x^3-5x^2+26x-21\right)\div\left(x^2+3x-4\right)=\boxed{x^2-2x+5 \text{ R } 3x-1}$$

We can check our division by multiplying back:

$$\begin{aligned}
P &= \left(x^2+3x-4\right)\left(x^2-2x+5+\frac{3x-1}{x^2+3x-4}\right)\\
&= \left(x^2+3x-4\right)\left(x^2-2x+5\right)+\left(x^2+3x-4\right)\frac{3x-1}{x^2+3x-4}\\
&= x^2\left(x^2-2x+5\right)+3x\left(x^2-2x+5\right)-4\left(x^2-2x+5\right)+3x-1\\
&= x^4-2x^3+5x^2+3x^3-6x^2+15x-4x^2+8x-20+3x-1=x^4+x^3-5x^2+26x-21
\end{aligned}$$

 Last revised: November 12, 2012