Sample Problems

Solve each of the following equations.

1.) $\sin x = -\frac{\sqrt{3}}{2}$ 2.) $\cos x = -\frac{1}{2}$ 3.) $\tan x = -1$ 4.) $\cos x = 2$ 5.) $\sin x = -1$

Practice Problems

1. Solve each of the following equations.

   a) $\tan x = -\frac{1}{\sqrt{3}}$ d) $\sin x = -\frac{1}{2}$ c) $\cos x = 1$

   b) $\sin x = 0$ e) $\sin x = -1.3$ f) $\cos x = \frac{\sqrt{3}}{2}$

2. Find the domain of each of the following functions.

   a) $f(x) = \tan x$ b) $g(x) = \cot x$ c) $t(x) = \frac{1}{\sin^2 x + \cos^2 x}$

Sample Problems- Answers

1.) $-\dfrac{\pi}{3}+2k\pi$ and $-\dfrac{2\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$ 2.) $x=\pm\dfrac{2\pi}{3}+2k\pi$ $k\in\mathbb{Z}$

3.) $x=-\dfrac{\pi}{4}+k\pi$, where $k\in\mathbb{Z}$ 4.) no solution 5.) $-\dfrac{\pi}{2}+2k\pi$ where $k\in\mathbb{Z}$

Practice Problems - Answers

1. a) $-\dfrac{1}{6}\pi+k\pi$ $k\in\mathbb{Z}$ b) $k\pi$ $k\in\mathbb{Z}$ c) $2k\pi$ $k\in\mathbb{Z}$

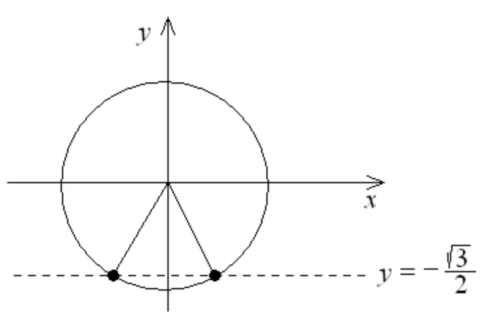
   d) $-\dfrac{1}{6}\pi+2k_1\pi$ and $\dfrac{7}{6}\pi+2k_2\pi$ $k_1,k_2\in\mathbb{Z}$ e) no solution f) $\pm\dfrac{\pi}{6}+2k\pi$ $k\in\mathbb{Z}$

2. a) $x\neq\dfrac{\pi}{2}+k\pi$ $k\in\mathbb{Z}$ b) $x\neq k\pi$ $k\in\mathbb{Z}$ c) $\mathbb{R}$

Sample Problems - Solutions

1.) Solve the equation $\sin x=-\dfrac{\sqrt{3}}{2}$.

Step 1. We draw a unit circle to find all solutions on it first. $\sin x=-\dfrac{\sqrt{3}}{2}$ means that we are looking for all points on it with second coordinate $-\dfrac{\sqrt{3}}{2}$. The two solutions on the unit circle are the intersections of the unit circle and the line $y=-\dfrac{\sqrt{3}}{2}$.



Step 2. We translate the solutions to two angles. In this case, $-60°$ and $-120°$ are two reasonable choices. ($240°$ and $300°$ are also correct).
Step 3. We algebraically state the result, which are the previously found two angles, and all co-terminal angles.

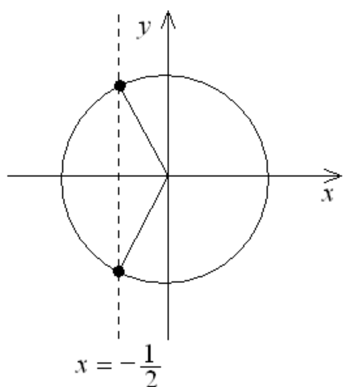
$$x = -60° + k \cdot 360° \quad \text{or} \quad x = -120° + k \cdot 360° \text{ where } \ k = 0, 1, -1, 2, -2, 3, -3, ...$$

Step 4. Convert the solutions to radians. Remember, $180° = \pi$ radians.

$$x = -\frac{\pi}{3} + 2k\pi \text{ or } x = -\frac{2\pi}{3} + 2k\pi \text{ where } \ k = 0, 1, -1, 2, -2, 3, -3, ...$$

2.) Solve the equation $\cos x = -\frac{1}{2}$.

Step 1. We draw a unit circle to find all solutions on it first. $\cos x = -\frac{1}{2}$ means that we are looking for all points on it with first coordinate $-\frac{1}{2}$. The two solutions on the unit circle are the intersections of the unit circle and the line $x = -\frac{1}{2}$.



Step 2. We translate the solutions to two angles. In this case, $120°$ and $-120°$ are two reasonable choices. ($120°$ and $240°$ are also correct).
Step 3. We algebraically state the result, which are the previously found two angles, and all co-terminal angles.

$$x = 120° + k \cdot 360° \text{ or } x = -120° + k \cdot 360° \text{ where } k \in \mathbb{Z}$$

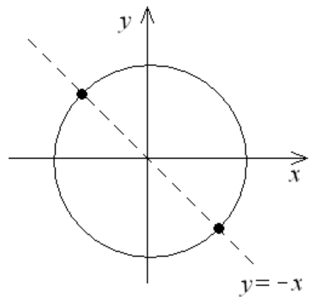
Notice that the new notation, $k \in \mathbb{Z}$ is a shorter way to write $k = 0, 1, -1, 2, -2, 3, -3, ...$In the same spirit, the result (unlike with $\sin x$) can be written as a single line, as

$$x = \pm 120° + k \cdot 360° \quad \text{where } k \in \mathbb{Z}$$

Step 4. Convert the solutions to radians. Remember, $180° = \pi$ radians.

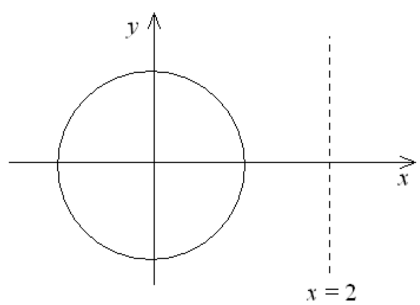
$$x = \pm\frac{2\pi}{3} + 2k\pi \qquad k \in \mathbb{Z}$$

3.) Solve the equation $\tan x = -1$.
Like in the case of sine and cosine, there will also be two solutions on the unit circle. Also, they will be the intersections of a line and the unit circle. $\tan x = -1$ is true when $x = -45°$. The second solution on the unit circle can be found by continuing the same straight line that connected $-45°$ with the origin. The solutions are now on the line of whose **slope** is -1.

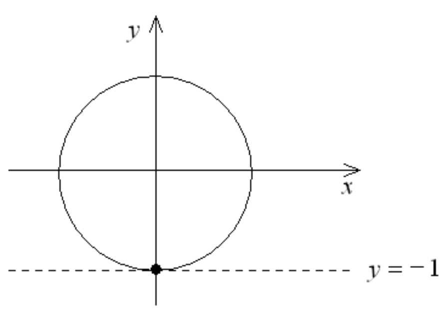


The solutions are $x = -45° + k \cdot 180°$ where $k \in \mathbb{Z}$. Or, in radians, $x = -\frac{\pi}{4} + k\pi$, where $k \in \mathbb{Z}$.

4.) Solve the equation $\cos x = 2$.
We draw a unit circle to find all solutions on it first. $\cos x = 2$ means that we are looking for all points on it with first coordinate 2. The solutions would be the intersections of the unit circle and the line $x = 2$. Since the circle and line do not intersect, there are no solutions to this equation.



5.) Solve the equation $\sin x = -1$.
We draw a unit circle to find all solutions on it first. $\sin x = -1$ means that we are looking for all points on it with second coordinate -1. There is only one solution on the unit circle because the line $y = -1$ is a tangent line.



The intersection point corresponds to $-90°$ (or $270°$).

$$x = -90° + k \cdot 360° \qquad k \in \mathbb{Z} \text{ or in radians: } \quad -\frac{\pi}{2} + 2k\pi \quad \text{where} \quad k \in \mathbb{Z}$$

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