Sample Problems

1. Find the exact value of $\sin\alpha$ if we know that α is in the fourth quadrant and $\cos\alpha = \frac{1}{3}$.

2. Suppose that x is in the third quadrant and $\sin x = -\frac{2}{5}$. Find the exact value of each of the following.

   a) $\cos x$ b) $\sec x$ c) $\tan x$

3. Find the exact value of $\cos\alpha$ where α is an acute angle with $\tan\alpha = \frac{1}{2}$.

4. If $\cot A = 2$, then what is the exact value of $\sin A$?

5. Simplify $\dfrac{\tan t + \sin(-t)\cos(-t)}{\tan t}$.

Practice Problems

1. Find the exact value of $\cos\alpha$ if we know that α is NOT in the fourth quadrant and $\sin\alpha = -\frac{3}{4}$.

2. Find the exact value of $\tan B$ if we know that $90° < B < 180°$ and $\cos B = -\frac{1}{5}$.

3. Suppose that x is in the second quadrant and $\cot x = -3$. Find the exact value of $\sin x$.

4. Suppose that x is in NOT the second quadrant and $\cot x = -3$. Find the exact value of $\sin x$.

Sample Problems - Answers

1.) $-\dfrac{\sqrt{8}}{3}$ 2.) a) $-\dfrac{\sqrt{21}}{5}$ b) $-\dfrac{5}{\sqrt{21}}$ c) $\dfrac{2}{\sqrt{21}}$ 3.) $\dfrac{2}{\sqrt{5}}$ 4.) $\pm\dfrac{1}{\sqrt{5}}$ 5.) $\sin^2 t$

Practice Problems - Answers

1.) $-\dfrac{\sqrt{7}}{4}$ 2.) $-2\sqrt{6}$ 3.) $\dfrac{1}{\sqrt{10}}$ 4.) $-\dfrac{1}{\sqrt{10}}$

Sample Problems - Solutions

1. Find the exact value of $\sin\alpha$ if we know that α is in the fourth quadrant and $\cos\alpha = \dfrac{1}{3}$.
Solution: We can solve for $\sin\alpha$ in the equation $\sin^2\alpha + \cos^2\alpha = 1$.

$$\begin{aligned} \sin^2\alpha + \cos^2\alpha &= 1 \\ \sin^2\alpha &= 1 - \cos^2\alpha \\ \sin\alpha &= \pm\sqrt{1-\cos^2\alpha} = \pm\sqrt{1-\left(\frac{1}{3}\right)^2} = \pm\sqrt{1-\frac{1}{9}} = \pm\sqrt{\frac{8}{9}} = \pm\frac{\sqrt{8}}{3} \end{aligned}$$

Since α is in the fourth quadrant, $\sin\alpha$ is negative. Thus $\sin\alpha = -\dfrac{\sqrt{8}}{3}$

2. Suppose that x is in the third quadrant and $\sin x = -\dfrac{2}{5}$. Find the exact value of each of the following.

a) $\cos x$
Solution: We can solve for $\cos x$ in the equation $\sin^2 x + \cos^2 x = 1$.

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \cos^2 x &= 1 - \sin^2 x \\ \cos x &= \pm\sqrt{1-\sin^2 x} = \pm\sqrt{1-\left(-\frac{2}{5}\right)^2} = \pm\sqrt{1-\frac{4}{25}} = \pm\sqrt{\frac{21}{25}} = \pm\frac{\sqrt{21}}{5} \end{aligned}$$

Since x is in the third quadrant, $\cos x$ is negative. Thus $\cos x = -\dfrac{\sqrt{21}}{5}$

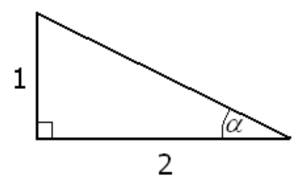
b) $\sec x$

Solution: $\sec x = \dfrac{1}{\cos x} = \dfrac{1}{-\dfrac{\sqrt{21}}{5}} = -\dfrac{5}{\sqrt{21}}$

c) $\tan x$

Solution: $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{-\dfrac{2}{5}}{-\dfrac{\sqrt{21}}{5}} = -\dfrac{2}{5}\left(-\dfrac{5}{\sqrt{21}}\right) = \dfrac{2}{\sqrt{21}}$

3. Find the exact value of $\cos\alpha$ where α is an acute angle with $\tan\alpha = \dfrac{1}{2}$.
Solution: We will see two different methods of solving for this problem.
Method 1. Since α is an acute angle, all trigonometric functions are positive. Now that we established the signs, let us draw a right triangle where $\tan\alpha = \dfrac{1}{2}$.



We can easily compute the hypotenuse of this triangle, using the Pythagorean theorem. The hypotenuse is $\sqrt{5}$ long. Then we can read the value of $\cos\alpha$ to be $\dfrac{2}{\sqrt{5}}$.
Method 2. Let us start with the Pythagorean identity.

$$\begin{aligned}
\sin^2\alpha + \cos^2\alpha &= 1 && \text{divide both sides by } \cos^2\alpha \\
\frac{\sin^2\alpha}{\cos^2\alpha} + \frac{\cos^2\alpha}{\cos^2\alpha} &= \frac{1}{\cos^2\alpha} \\
\tan^2\alpha + 1 &= \frac{1}{\cos^2\alpha} && \text{take the reciprocal of both sides} \\
\frac{1}{\tan^2\alpha + 1} &= \cos^2\alpha && \text{square-root property} \\
\pm\sqrt{\frac{1}{\tan^2\alpha + 1}} &= \cos\alpha
\end{aligned}$$

Since α is an acute angle, $\cos\alpha$ is the positive solution:

$$\cos\alpha = \sqrt{\frac{1}{\tan^2\alpha + 1}} = \sqrt{\frac{1}{\left(\dfrac{1}{2}\right)^2 + 1}} = \sqrt{\frac{1}{\dfrac{5}{4}}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

4. If $\cot A = 2$, then what is the exact value of $\sin A$? $\pm\dfrac{1}{\sqrt{5}}$

   Solution:

$$\begin{aligned}
\sin^2 A + \cos^2 A &= 1 && \text{divide by } \sin^2 A \\
\frac{\sin^2 A}{\sin^2 A} + \frac{\cos^2 A}{\sin^2 A} &= \frac{1}{\sin^2 A} \\
1 + \cot^2 A &= \frac{1}{\sin^2 A} && \cot A = 2 \\
1 + 2^2 &= \frac{1}{\sin^2 A} \\
5 &= \frac{1}{\sin^2 A} \\
\sin^2 A &= \frac{1}{5} \\
\sin A &= \pm\frac{1}{\sqrt{5}}
\end{aligned}$$

5. Simplify $\dfrac{\tan t + \sin(-t)\cos(-t)}{\tan t}$

   Solution: Since $\sin(-t) = -\sin t$ and $\cos(-t) = \cos t$, we can simplify the expression as

$$\frac{\tan t + \sin(-t)\cos(-t)}{\tan t} = \frac{\tan t - \sin t\cos t}{\tan t} = \frac{\dfrac{\sin t}{\cos t} - \sin t\cos t}{\dfrac{\sin t}{\cos t}} = \frac{\cos t}{\cos t} \cdot \frac{\dfrac{\sin t}{\cos t} - \sin t\cos t}{\dfrac{\sin t}{\cos t}}$$

$$= \frac{\cos t\left(\dfrac{\sin t}{\cos t} - \sin t\cos t\right)}{\cos t\left(\dfrac{\sin t}{\cos t}\right)} = \frac{\sin t - \sin t\cos^2 t}{\sin t} = \frac{\sin t\,(1 - \cos^2 t)}{\sin t}$$

$$= \frac{\sin t\sin^2 t}{\sin t} = \frac{\sin^3 t}{\sin t} = \sin^2 t$$