

Sample Problems

Find the exact value of each of the following and simplify.

1. $2 \sin 60^\circ - 5 \tan 30^\circ \cot 45^\circ$

2. $2 \sin 60^\circ - 3 \cos 45^\circ + \tan 30^\circ$

3. $4 \cos 30^\circ - \sin 45^\circ + \tan 60^\circ$

4. $\sin 30^\circ \cos 45^\circ \tan 60^\circ - 2 \sec 60^\circ + 3 \csc 45^\circ \tan 30^\circ \cos 60^\circ$

5. $\frac{\sin 60^\circ - 1}{\cos 30^\circ + 1}$

Practice Problems

Find the exact value of each of the following and simplify.

1. $\sin 45^\circ - 2 \cos 60^\circ \tan 30^\circ - \csc 45^\circ$

2. $3 \tan 60^\circ - 3 \cos 45^\circ \sec 60^\circ - \tan 30^\circ \tan 60^\circ \csc 45^\circ$

3. $\frac{\cos 45^\circ - \cot 45^\circ}{\sin 45^\circ + \tan 45^\circ}$

4. $\sin^2 30^\circ - \cos 45^\circ \tan^2 60^\circ - 2 \cot 60^\circ \tan 60^\circ$

5. $3 \sec 30^\circ - \cos 60^\circ \sin 60^\circ$

6. $\frac{\tan 60^\circ - \tan 30^\circ}{\tan 60^\circ + \tan 30^\circ}$

7. $\frac{\sin 30^\circ \cos 30^\circ \tan 30^\circ \cot 30^\circ}{\sin 45^\circ \cos 45^\circ \tan 45^\circ \cot 45^\circ}$

8. $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$

9. $\frac{(\sin 30^\circ)(\cos 30^\circ) - (\tan 30^\circ) \cot 30^\circ}{(\sin 45^\circ)(\cos 45^\circ) - (\tan 45^\circ) \cot 45^\circ}$

Sample Problems - Answers

$$\begin{array}{lll} 1.) \quad -\frac{2\sqrt{3}}{3} & 2.) \quad \frac{8\sqrt{3}-9\sqrt{2}}{6} \text{ or } \frac{4\sqrt{3}}{3} - \frac{3\sqrt{2}}{2} & 3.) \quad 3\sqrt{3} - \frac{\sqrt{2}}{2} \text{ or } \frac{6\sqrt{3}-\sqrt{2}}{2} \\ 4.) \quad \frac{3}{4}\sqrt{6} - 4 \text{ or } \frac{3\sqrt{6}-16}{4} & 5.) \quad -7 + 4\sqrt{3} \end{array}$$

Practice Problems - Answers

$$\begin{array}{lllllll} 1.) \quad -\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3} & 2.) \quad 3\sqrt{3} - 4\sqrt{2} & 3.) \quad 2\sqrt{2} - 3 & 4.) \quad -\frac{3\sqrt{2}}{2} - \frac{7}{4} & 5.) \quad \frac{7}{4}\sqrt{3} & 6.) \quad \frac{1}{2} & 7.) \quad \frac{\sqrt{3}}{2} \\ 8.) \quad \sqrt{3} & 9.) \quad 2 - \frac{\sqrt{3}}{2} \end{array}$$

Sample Problems - Solutions

Find the exact value of each of the following and simplify.

$$1. \quad 2 \sin 60^\circ - 5 \tan 30^\circ \cot 45^\circ = -\frac{2\sqrt{3}}{3}$$

Solution: We first obtain the exact values of the trigonometric expressions. Using the famous right triangles, we obtain that

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \tan 30^\circ = \frac{1}{\sqrt{3}} \quad \text{and} \quad \cot 45^\circ = 1$$

Because we will be adding and subtracting these expressions, it is best if we rationalize them first.

$$\tan 30^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$2 \sin 60^\circ - 5 \tan 30^\circ \cot 45^\circ = 2 \cdot \frac{\sqrt{3}}{2} - 5 \cdot \frac{\sqrt{3}}{3} \cdot 1 = \sqrt{3} - \frac{5\sqrt{3}}{3}$$

We will then bring these to the common denominator

$$\sqrt{3} - \frac{5\sqrt{3}}{3} = \frac{3\sqrt{3}}{3} - \frac{5\sqrt{3}}{3} = \frac{3\sqrt{3} - 5\sqrt{3}}{3} = \boxed{\frac{-2\sqrt{3}}{3}}$$

$$2. \quad 2 \sin 60^\circ - 3 \cos 45^\circ + \tan 30^\circ$$

Solution: We first obtain the exact values of the trigonometric expressions. Using the famous right triangles, we obtain that

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \text{and} \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Because we will be adding and subtracting these expressions, it is best if we rationalize them first.

$$\cos 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad \text{and} \quad \tan 30^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$2 \sin 60^\circ - 3 \cos 45^\circ + \tan 30^\circ = 2 \cdot \frac{\sqrt{3}}{2} - 3 \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{3} = \sqrt{3} - \frac{3\sqrt{2}}{2} + \frac{\sqrt{3}}{3}$$

We will then bring these to the common denominator, which is 6 and if there are like terms, we combine them.

$$\begin{aligned} \sqrt{3} - \frac{3\sqrt{2}}{2} + \frac{\sqrt{3}}{3} &= \frac{6\sqrt{3}}{6} - \frac{9\sqrt{2}}{6} + \frac{2\sqrt{3}}{6} \\ &= \frac{6\sqrt{3} - 9\sqrt{2} + 2\sqrt{3}}{6} = \boxed{\frac{8\sqrt{3} - 9\sqrt{2}}{6}} \end{aligned}$$

Note: $\frac{8\sqrt{3} - 9\sqrt{2}}{6} = \frac{8\sqrt{3}}{6} - \frac{9\sqrt{2}}{6} = \boxed{\frac{4\sqrt{3}}{3} - \frac{3\sqrt{2}}{2}}$ is also perfectly acceptable as final answer.

$$3. \quad 4 \cos 30^\circ - \sin 45^\circ + \tan 60^\circ$$

Solution: We first obtain the exact values of the trigonometric expressions. Using the famous right triangles, we obtain that

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \sin 45^\circ = \frac{1}{\sqrt{2}} \quad \text{and} \quad \tan 60^\circ = \sqrt{3}$$

Because we will be adding and subtracting these expressions, it is best if we rationalize them first.

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$4 \cos 30^\circ - \sin 45^\circ + \tan 60^\circ = 4 \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} + \sqrt{3} = 2\sqrt{3} - \frac{\sqrt{2}}{2} + \sqrt{3} = \boxed{3\sqrt{3} - \frac{\sqrt{2}}{2}}$$

Note: We can also bring these to the common denominator.

$$3\sqrt{3} - \frac{\sqrt{2}}{2} = \frac{6\sqrt{3}}{2} - \frac{\sqrt{2}}{2} = \boxed{\frac{6\sqrt{3} - \sqrt{2}}{2}}$$

is also perfectly acceptable.

$$4. \sin 30^\circ \cos 45^\circ \tan 60^\circ - 2 \sec 60^\circ + 3 \csc 45^\circ \tan 30^\circ \cos 60^\circ = \frac{3}{4}\sqrt{6} - 4$$

Solution: We first obtain the exact values of the trigonometric expressions. Using the famous right triangles, we obtain that

$$\sin 30^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \tan 60^\circ = \sqrt{3} \quad \sec 60^\circ = 2 \quad \csc 45^\circ = \sqrt{2} \quad \tan 30^\circ = \frac{1}{\sqrt{3}} \quad \text{and} \quad \cos 60^\circ = \frac{1}{2}$$

Because we will be adding and subtracting these expressions, it is best if we rationalize them first.

$$\cos 45^\circ = \frac{\sqrt{2}}{2} \quad \text{and} \quad \tan 30^\circ = \frac{\sqrt{3}}{3}$$

$$\begin{aligned} \sin 30^\circ \cos 45^\circ \tan 60^\circ - 2 \sec 60^\circ + 3 \csc 45^\circ \tan 30^\circ \cos 60^\circ &= \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \cdot \sqrt{3} - 2 \cdot 2 + 3 \cdot \sqrt{2} \cdot \frac{\sqrt{3}}{3} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} - 4 + \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{4} - 4 + \frac{\sqrt{6}}{2} = \frac{\sqrt{6}}{4} - 4 + \frac{2\sqrt{6}}{4} = \boxed{\frac{3\sqrt{6}}{4} - 4} \end{aligned}$$

Note: We can also bring these to the common denominator.

$$\frac{3\sqrt{6}}{4} - 4 = \frac{3\sqrt{6}}{4} - \frac{16}{4} = \boxed{\frac{3\sqrt{6} - 16}{4}}$$

is also perfectly acceptable.

$$5. \frac{\sin 60^\circ - 1}{\cos 30^\circ + 1} = -7 + 4\sqrt{3}$$

Solution: We first obtain the exact values of the trigonometric expressions. Using the famous right triangles, we obtain that

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \text{and} \quad \cos 30^\circ = \frac{\sqrt{3}}{2} \quad \text{and} \quad \text{so} \quad \frac{\sin 60^\circ - 1}{\cos 30^\circ + 1} = \frac{\frac{\sqrt{3}}{2} - 1}{\frac{\sqrt{3}}{2} + 1}$$

To clear the denominators, we multiply both numerator and denominator by 2

$$\frac{\frac{\sqrt{3}}{2} - 1}{\frac{\sqrt{3}}{2} + 1} = \frac{2 \left(\frac{\sqrt{3}}{2} - 1 \right)}{2 \left(\frac{\sqrt{3}}{2} + 1 \right)} = \frac{\sqrt{3} - 2}{\sqrt{3} + 2}$$

We rationalize our result:

$$\frac{\sqrt{3} - 2}{\sqrt{3} + 2} = \frac{\sqrt{3} - 2}{\sqrt{3} + 2} \cdot \frac{\sqrt{3} - 2}{\sqrt{3} - 2} = \frac{(\sqrt{3} - 2)^2}{3 - 4} = \frac{3 - 4\sqrt{3} + 4}{-1} = \frac{7 - 4\sqrt{3}}{-1} = - (7 - 4\sqrt{3}) = \boxed{-7 + 4\sqrt{3}}$$