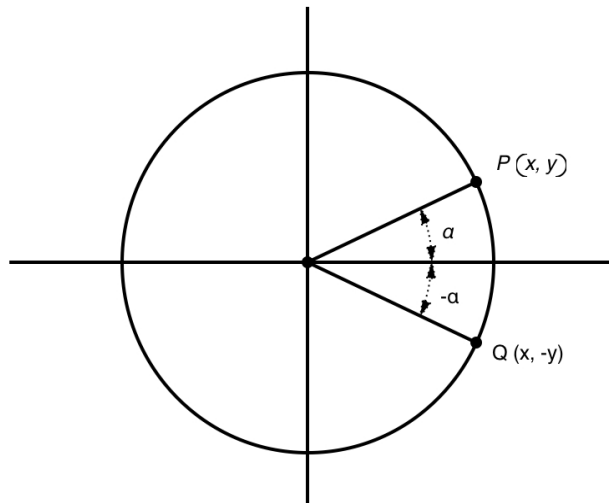


Let  $\alpha$  be an angle denoted by  $P(x, y)$  on the unit circle. If  $\alpha$  is measured from the positive part of the  $x$ -axis, counterclockwise, then by definition,

$$\cos \alpha = x \text{ and } \sin \alpha = y$$

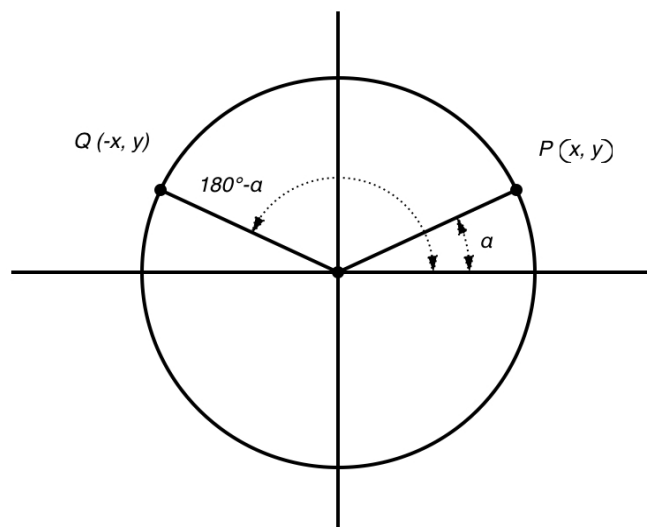
Case 1. Reflect  $P$  to the  $x$ -axis to obtain point  $Q(x, -y)$ . Then the angle associated by  $Q$  is  $-\alpha$ .

$$\begin{aligned} \sin(-\alpha) &= -\sin \alpha \\ \cos(-\alpha) &= \cos \alpha \\ \tan(-\alpha) &= \frac{\sin(-\alpha)}{\cos(-\alpha)} = \frac{-\sin \alpha}{\cos \alpha} = -\tan \alpha \end{aligned}$$



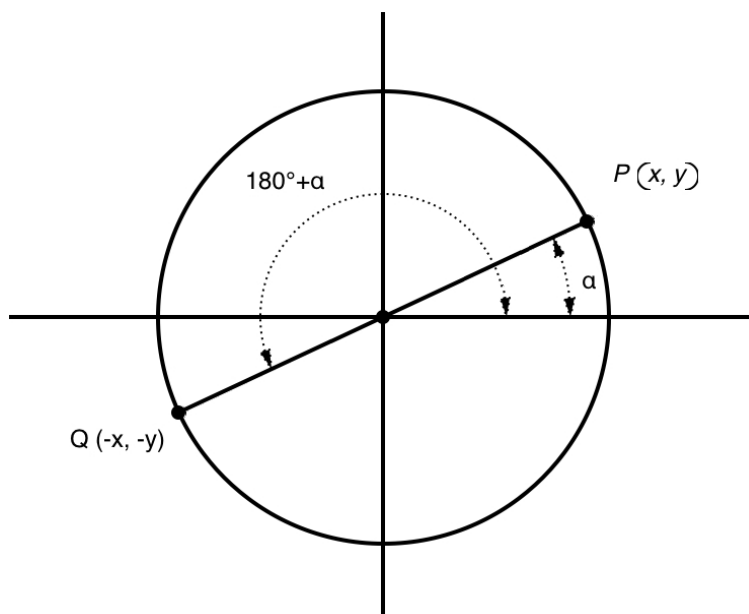
Case 2. Reflect  $P$  to the  $y$ -axis to obtain point  $Q(-x, y)$ . Then the angle associated by  $Q$  is  $180^\circ - \alpha$ . In other words, these two angles are supplements.

$$\begin{aligned} \sin(180^\circ - \alpha) &= \sin \alpha \\ \cos(180^\circ - \alpha) &= -\cos \alpha \\ \tan(180^\circ - \alpha) &= \frac{\sin(180^\circ - \alpha)}{\cos(180^\circ - \alpha)} = \frac{\sin \alpha}{-\cos \alpha} = -\tan \alpha \end{aligned}$$



Case 3. Reflect  $P$  to the origin to obtain point  $Q(-x, -y)$ . Then the angle associated by  $Q$  is  $\alpha + 180^\circ$ .

$$\begin{aligned}\sin(\alpha + 180^\circ) &= -\sin \alpha \\ \cos(\alpha + 180^\circ) &= -\cos \alpha \\ \tan(\alpha + 180^\circ) &= \frac{\sin(\alpha + 180^\circ)}{\cos(\alpha + 180^\circ)} = \frac{-\sin \alpha}{-\cos \alpha} = \tan \alpha\end{aligned}$$



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