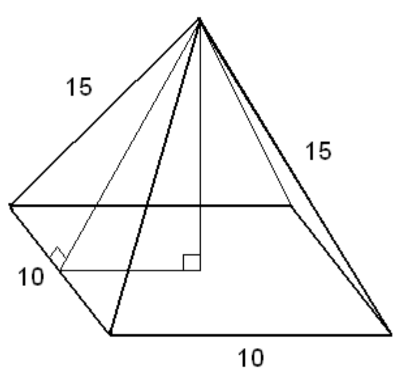
Sample Problems

1. A surveyor wishes to determine the height of a building. He measures a distance of 1500 feet along the ground from the building, and there he determines the angle of elevation to the top of the building. This angle is 24°. How tall is the building? Present your answer as
   a) an exact value b) as an approximation, accurate up to five decimal places.
2. A ladder 9 m long is placed against a building. The angle between the ladder and the ground is 65°. How high up is the top of the ladder? Present your answer as
   a) an exact value b) as an approximation, accurate up to five decimal places.
3. Compute the angles (up to five decimal places) in a triangle with sides 18 m, 13 m, and 13 m long.
4. There is a 10-mile pathway towards the ocean, perpendicular to the beach. Two runners agree to race on this path. The first runner runs the path with a perfect accuracy. The second runner misses the perpendicular path by 1° to the right.
   a) How far are apart are the endpoints of the runners' paths?
   b) How much more did the second runner run than the first runner, due to the inaccuracy of the path?
5. From a hot air balloon that is 4000 ft high, the angles of depression to two houses, in line with the balloon, are 47° and 32°. How far apart are the houses?
6. Find the area of a regular polygon of 10 sides, inscribed in a circle of radius 5 m.
7. Find the angle that the straight line $y = \dfrac{2}{3}x + 5$ forms with the positive half of the x-axis.
8. Consider a circle with radius 4 units. The distance between the center of the circle and a point P is 12 units. Compute the angle that is formed by the two tangent lines drawn to the circle from P. Present your answer as
   a) exact value b) an approximation, accurate up to four decimal places.
9. The picture below shows a straight pyramid with a square base. The sides of the base are 10 in long. The other edges are 15 in long. Find the angle between the base and a triangular face.



10. We are driving toward a tower. The angle of elevation to the top of the tower is 28°. Then we drive 550 ft toward the tower. Now the angle of elevation is 40°. How tall is the tower?

Practice Problems

1. A surveyor wishes to determine the height of a building. He measures a distance of 80 meter along the ground from the building, and there he determines the angle of elevation to the top of the building. This angle is $18°$. How tall is the building? Present your answer as
   a) an exact value b) as an approximation, accurate up to five decimal places.

2. A ladder 5 m long is placed against a building. The angle between the ladder and the building is $15°$. How high up is the top of the ladder? Present your answer as
   a) an exact value b) as an approximation, accurate up to five decimal places.

3. Compute the angles (up to five decimal places) in a triangle with sides 10 m, 21 m, and 21 m long.

4. There is a 26-mile pathway towards the ocean, perpendicular to the beach. Two runners agree to race on this path. The first runner runs the path with a perfect accuracy. The second runner misses the perpendicular path by $1.5°$ to the right.
   a) How far are apart are the endpoints of the runners' paths?
   b) How much more did the second runner run than the first runner, due to the inaccuracy of the path?

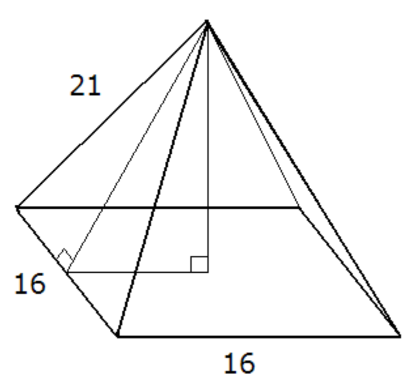
5. From a hot air balloon that is 500 m high, the angles of depression to two houses, in line with the balloon, are $38°$ and $49°$. How far apart are the houses?

6. A regular polygon of 12 sides is written into a circle of radius 5 ft. Find the area of the polygon.

7. Find the angle that is formed by the positive part of the x-axis and the straight line $2x - 5y = 1$.

8. A point P is 10 cm away from the center C of a circle with radius 7 cm. Compute the angle that is formed by the two tangent lines drawn to the circle from P. Present your answer as
   a) exact value b) an approximation, accurate up to four decimal places.

9. The picture below shows a straight pyramid with a square base. The sides of the base are 16 in long. The other edges are 21 in long. Find the angle between the base and a triangular face.



10. The angle of elevation to the top of a building changes from $12°$ to $25°$ as the observer travels 200 m towards the building. How tall is the building?

Sample Problems - Answers

1.) a) $1500 \tan 24^\circ$ ft b) $667.843\,03$ ft 2.) a) $9 \sin 65^\circ$ m b) $8.156\,77$ m

3.) $\alpha = \beta \approx 46.18694^\circ$ and $\gamma \approx 87.626\,12^\circ$

4.) a) 0.17455 mile apart b) the second runner ran 0.00152 mile more than the first runner

5.) $10131.398\,461$ ft 6.) $73.473\,16\,\text{m}^2$ 7.) 33.690° 8.) a) $2\sin^{-1}\left(\frac{1}{3}\right)$ b) $38.942\,4^\circ$

9.) $69.295\,2^\circ$ 10.) 798.290 ft

Practice Problems - Answers

1.) a) $80 \tan 18^\circ$ m b) $25.993\,58$ m 2.) a) $5\cos 15^\circ$ m b) $4.829\,63$ m

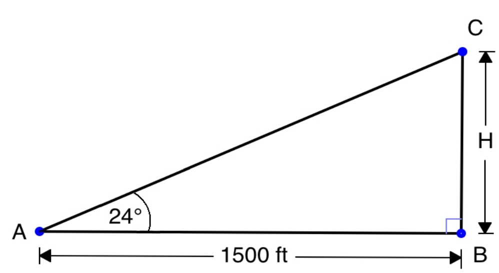
3.) $76.225\,853^\circ$ $76.225\,853^\circ$ and $27.548\,294^\circ$ 4.) a) $0.680\,834$ mile b) $0.008\,9$ mile more

5.) $1074.614\,19$ m 6.) $75\,\text{ft}^2$ 7.) 21.801° 8.) a) $2\sin^{-1}\left(\frac{7}{10}\right)$ b) $88.854\,0^\circ$

9.) $65.668\,15^\circ$ 10.) $78.121\,2$ m

Sample Problems - Solutions

1. A surveyor wishes to determine the height of a building. He measures a distance of 1500 feet along the ground from the building, and there he determines the angle of elevation to the top of the building. This angle is 24°. How tall is the building?
   Solution: Consider the right triangle shown on the picture below. We denote the height of the building by H.



There is one trigonometric statement connecting $24°$, 1500 feet, and H, and that is $\tan 24°$ stated on the right triangle ABC.

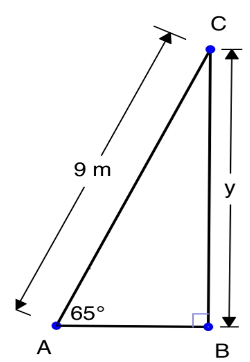
$$\begin{aligned} \tan 24° &= \frac{H}{1500\,\text{ft}} && \text{multiply by } 1500\,\text{ft} \\ (1500\,\text{ft})\tan 24° &= H \\ H &= 1500\tan 24°\,\text{ft} && \text{exact value} \end{aligned}$$

To compute an approximate value, we enter $1500 \cdot \tan 24$ into our calculator. We need to be careful to enter exactly what we mean (What is the danger in entering $\tan 24 \cdot 1500$?) Also, make sure that your calculator is in degree mode and not in radian or gradient mode. The calculator shows

$$1500\tan 24° \approx 667.843\,027\,963$$

which we present as $667.843\,03$ after correctly rounding.

2. A ladder 9 m long is placed against a building. The angle between the ladder and the ground is $65°$. How high up is the top of the ladder? Present your answer as
a) an exact value b) as an approximation, accurate up to five decimal places.
Solution: Consider the right triangle shown on the picture below. We denote the height of the top of the ladder by y.

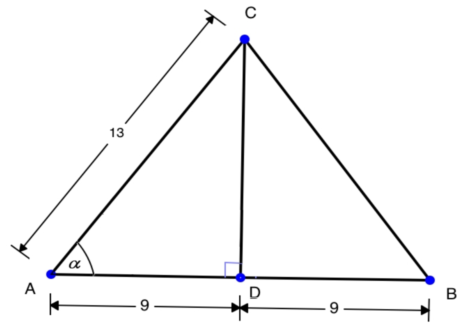


There is one trigonometric statement connecting $65°$, 9 meter, and y, and that is $\sin 65°$ stated on the right triangle ABC.

$$\begin{aligned} \sin 65° &= \frac{y}{9\,\text{m}} && \text{multiply by } 9\,\text{m} \\ 9\sin 65°\,\text{m} &= y && \text{this is the exact value} \end{aligned}$$

We enter this into the calculator and obtain the approximation $8.156\,77$ m

3. Compute the angles (up to five decimal places) in a triangle with sides 18 m, 13 m, and 13 m long.
Solution: Consider the picture below, where $AC = BC = 13$ m. Since this triangle isosceles, line CD is a perpendicular bisector of line segment AB. Thus we have a right triangle ADC, with sides $AC = 13$ m and $AD = 9$ m.



We can now compute α

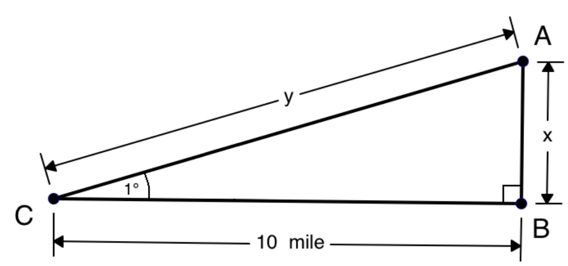
$$\cos\alpha = \frac{9}{13} \quad \Longrightarrow \quad \alpha = \cos^{-1}\left(\frac{9}{13}\right) \approx 46.18694^\circ$$

(Notice that the notations $\cos^{-1}$ and arccos represent the same thing. You will frequently see both notations.) Since the triangle is isosceles, $\beta = \alpha \approx 46.18694^\circ$. Now

$$\begin{aligned} \alpha + \beta + \gamma &= 180^\circ \\ \gamma &= 180^\circ - (\alpha + \beta) \approx 180^\circ - 2\,(46.18694^\circ) = 87.626\,12^\circ \end{aligned}$$

Thus $\alpha = \beta \approx 46.18694^\circ$ and $\gamma \approx 87.626\,12^\circ$.

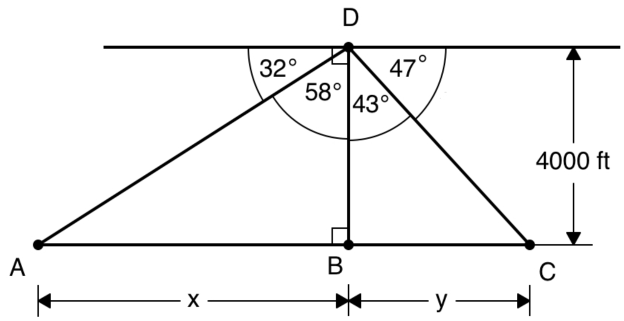
4. There is a 10-mile pathway towards the ocean, perpendicular to the beach. Two runners agree to race on this path. The first runner runs the path with a perfect accuracy. The second runner misses the perpendicular path by 1° to the right.
a) How far are apart are the endpoints of the runners' paths?
b) How much more did the second runner run than the first runner, due to the inaccuracy of the path?
Solution: Consider the right triangle below. Side segment BC represent the correct path, while side AC is the longer path. Side AB is the distance between the endpoints of the two paths.



$$\begin{aligned} \tan 1^\circ &= \frac{x}{10\,\text{mi}} \quad \Longrightarrow \quad x = (10\,\text{mi})\tan 1^\circ \approx 0.17455\,\text{mi} \\ \cos 1^\circ &= \frac{10\,\text{mi}}{y} \quad \Longrightarrow \quad y = \frac{10\,\text{mi}}{\cos 1^\circ} \approx 10.00152\,\text{mi} \end{aligned}$$

a) the endpoints of the paths are 0.017455 mile apart.
b) the second runner ran 000152 mile more than the first runner

5. From a hot air balloon that is 4000 ft high, the angles of depression to two houses, in line with the balloon, are 47° and 32°. How far apart are the houses?
Solution: Consider the picture below. Let D denote the baloon, and A and C denote the two houses. Point B is the point between the houses above which the baloon is located.



We first compute the angles inside the triangles as shown on the picture above. If the angle of elevation is 32°, then the other angle is its complement, $90^\circ - 32^\circ = 58^\circ$. We similarly obtain 58°. We can now compute x and y using the right triangles ABD and BCD.

$$\tan 58^\circ = \frac{x}{4000\,\text{ft}} \text{ from triangle } ABD \text{ and } \tan 43^\circ = \frac{y}{4000\,\text{ft}} \text{ from triangle } BCD$$

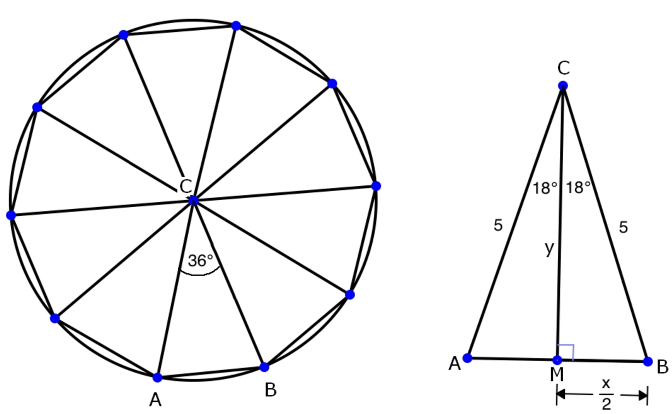
The distance between the houses is $x + y$. We solve these equations for x and y and then add.

$$\begin{aligned} \tan 58^\circ &= \frac{x}{4000\,\text{ft}} &\implies& \quad x = 4000 \tan 58^\circ \text{ ft} \\ \tan 43^\circ &= \frac{y}{4000\,\text{ft}} &\implies& \quad y = 4000 \tan 43^\circ \text{ ft} \end{aligned}$$

$$x + y = 4000 \tan 58^\circ \text{ ft} + 4000 \tan 43^\circ \text{ ft} = 4000 \left(\tan 58^\circ + \tan 43^\circ\right) \text{ ft} \approx 10131.398\,461 \text{ ft}$$

Caution! $\tan 58^\circ + \tan 43^\circ$ is not the same as $\tan\left(58^\circ + 43^\circ\right)$. It is very important to enter the expressions accurately.

6. Find the area of a regular polygon of 10 sides, inscribed in a circle of radius 5 m.
Solution: We draw a regular polygon of ten sides into a circle. We then connect each vertex with the center of the circle. We will look at one of the ten identical triangles that make up the polygon.



Consider the isosceles triangle ABC with an angle of $\dfrac{360^\circ}{10} = 36^\circ$ at the center. This triangle does not have a right angle. So we split the triangle in two identical parts by the height drawn to the base. We will find the area of one triangle and then multiply that area by 10 since 10 such triangles make up the polygon. Let us label the base AB by x, and the height CM by y. Then CBM triangle is a right triangle, with $MCB\angle = 18^\circ$, and the hypotenuse is $5\,\text{m}$.

$$\sin 18^\circ = \frac{\frac{x}{2}}{5\,\text{m}} = \frac{x}{10\,\text{m}} \qquad \Longrightarrow \qquad x = (10\,\text{m})\sin 18^\circ$$

To find y:

$$\cos 18^\circ = \frac{y}{5\,\text{m}} \qquad \Longrightarrow \qquad y = (5\,\text{m})\cos 18^\circ$$

Thus the triangle AOB has area

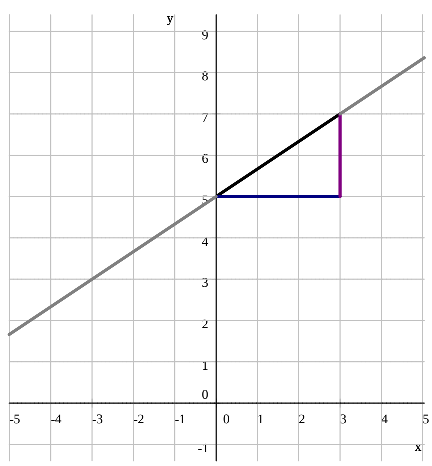
$$A = \frac{1}{2}xy = \frac{1}{2}\left(10\sin 18^\circ\,\text{m}\right)\left(5\cos 18^\circ\,\text{m}\right) = 25\sin 18^\circ\cos 18^\circ\,\text{m}^2$$

Thus the polygon's area is

$$10A = 10\left(25\sin 18^\circ\cos 18^\circ\,\text{m}^2\right) = 250\sin 18^\circ\cos 18^\circ\,\text{m}^2 \approx 73.473\,16\,\text{m}^2$$

7. Find the angle that the straight line $y = \dfrac{2}{3}x + 5$ forms with the positive half of the x-axis.
   Solution: Let α denote the angle between the line and the positive part of the $x-$axis. Consider that the slope means rise over run, i.e.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{3}$$



In the context of trigonometry, the slope will become $\tan\alpha$. Thus we need to enter into the calculator

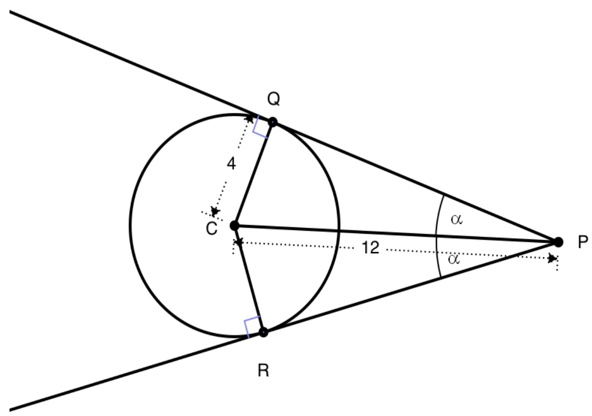
$$\tan^{-1}(2 \div 3) =$$

and be careful that the calculator is set to degrees. The answer is 33.690°.

8. Consider a circle with radius 4 units. The distance between the center of the circle and a point P is 12 units. Compute the angle that is formed by the two tangent lines drawn to the circle from P. Present your answer as

a) exact value b) an approximation, accurate up to four decimal places.

Solution: Let Q and R denote the points of tangency. Let us also connect the center of the circle C with P.



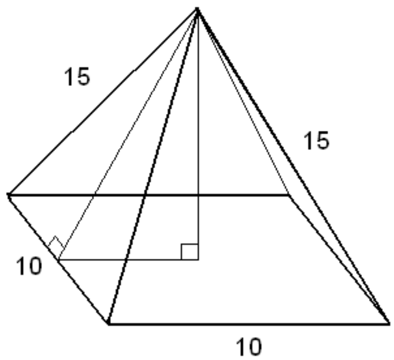
We need to notice two things. First, the picture is completely symmetrical to the line CP. Thus the angles QPC and CPR have the same measure. They are both denoted by α, and the angle that we have to compute is simply 2α. We will compute α using the triangle PQC. The second thing we need to notice is that this triangle is a right triangle because the tangent line of a circle is always perpendicular to the radius drawn to the point of tangency. Thus angle $PQC = 90°$. We now easily compute α using right triangle trigonometry:

$$\sin\alpha = \frac{4}{12} = \frac{1}{3} \qquad \Longrightarrow \qquad \alpha = \sin^{-1}\left(\frac{1}{3}\right)$$

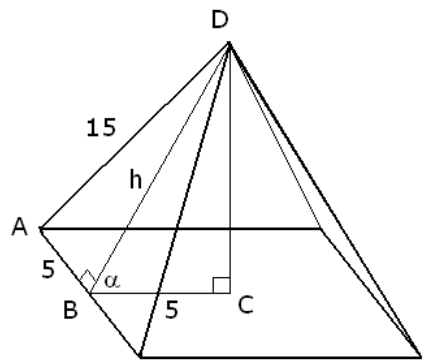
The angle we must compute is twice α and so the answer is, as an exact value, $2\alpha = 2\sin^{-1}\left(\frac{1}{3}\right)$ or, using the other type of notation, $2\alpha = 2\arcsin\left(\frac{1}{3}\right)$. To compute the approximate value, we enter this into the calculator and round to the requested accuracy. Make sure the calculator is set in degree mode.

$$2\alpha = (2)\arcsin\left(\frac{1}{3}\right) \approx 38.942\,4°$$

9. The picture below shows a straight pyramid with a square base. The sides of the base are 10 in long. The other sides are 15 in long. Find the angle between the base and a triangular face.



Solution: Let us first label the points and sides we will use as shown below. We need to find angle $\angle DBC = \alpha$.



We compute the height h of a face by the Pythagorean theorem stated for triangle ABD.

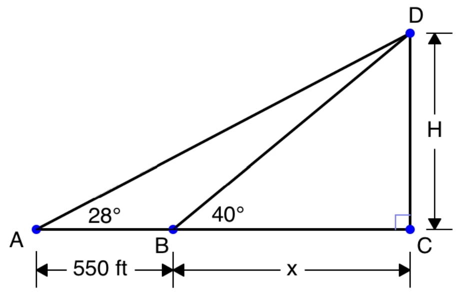
$$5^2 + h^2 = 15^2 \qquad \Longrightarrow \qquad h = \sqrt{200} = 10\sqrt{2}$$

Now

$$\cos\alpha = \frac{\overline{BC}}{\overline{BD}} = \frac{5}{h} = \frac{5}{10\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

and so $\alpha = \cos^{-1}\left(\frac{1}{2\sqrt{2}}\right) \approx 69.295\,19^\circ$

10. We are driving toward a tower. The angle of elevation is $28°$. Then we drive 550 ft toward the tower. Now the angle of elevation is $40°$. How tall is the tower?
Solution: Based on the picture below, we can write a system of linear equations.



$$\text{in triangle } ACD \qquad \tan 28° = \frac{H}{x+550}$$
$$\text{in triangle } BCD \qquad \tan 40° = \frac{H}{x}$$

In this system, there are two unknowns, H and x. We will use substitution to solve the system. It is a bit unusual, but common in trigonometry to symbolically carry the trigonometric expressions and only to substitute approximate values into them in the last line.

$$\tan 28° = \frac{H}{x+550} \implies H = \tan 28° (x+550)$$
$$\tan 40° = \frac{H}{x} \implies H = \tan 40° x$$

$$\begin{aligned}
\tan 28° (x+550) &= \tan 40° x \\
x \tan 28° + 550 \tan 28° &= x \tan 40° \\
550 \tan 28° &= x \tan 40° - x \tan 28° \\
550 \tan 28° &= x (\tan 40° - \tan 28°) \\
\frac{550 \tan 28°}{\tan 40° - \tan 28°} &= x
\end{aligned}$$

$$H = \tan 40° x = \tan 40° \left(\frac{550 \tan 28°}{\tan 40° - \tan 28°} \right) = \frac{550 \tan 40° \tan 28°}{\tan 40° - \tan 28°} \approx 798.289\,78 \ \ (\text{ft})$$

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