

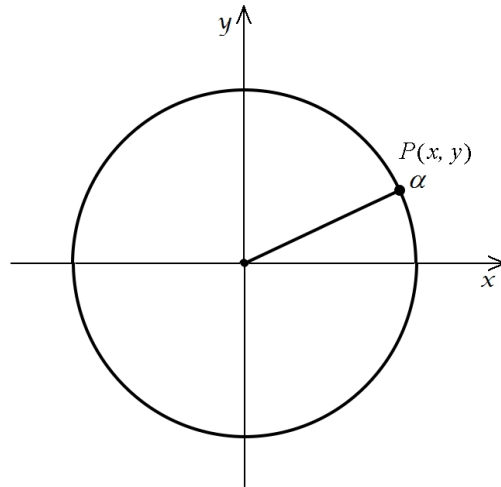
## The Unit Circle Definition of Trigonometric Functions

Let  $\alpha$  be **any** rotational angle. (It could be negative, or greater than  $180^\circ$ , or even greater than  $360^\circ$ .) We will define all six trigonometric functions of  $\alpha$  as follows.

**Step 1.** Draw a unit circle and draw a ray representing  $\alpha$ .

**Step 2.** For any angle  $\alpha$ , this ray intersects the unit circle in a single point  $P(x, y)$ .

**Step 3.** We define the trigonometric function values of  $\alpha$  as:



$$\cos \alpha = x$$

$$\sin \alpha = y$$

$$\tan \alpha = \frac{y}{x}$$

$$\sec \alpha = \frac{1}{x}$$

$$\csc \alpha = \frac{1}{y}$$

$$\cot \alpha = \frac{x}{y}$$

$\cos \alpha$  and  $\sin \alpha$  always exist. For the other four trigonometric functions, we will need to worry about division by zero. Since  $\tan \alpha = \frac{y}{x}$ , it will be defined when  $x \neq 0$ . Similarly,  $\sec \alpha = \frac{1}{x}$  is defined when  $x \neq 0$  and  $\csc \alpha = \frac{1}{y}$  and  $\cot \alpha = \frac{x}{y}$  will be defined if  $y \neq 0$ .

### Practice Problems

In case of each of the following angles, graph the angle in the unit circle and state its sine, cosine, and tangent.

1.  $180^\circ$

3.  $210^\circ$

5.  $270^\circ$

7.  $90^\circ$

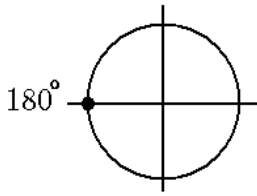
2.  $-60^\circ$

4.  $135^\circ$

6.  $-135^\circ$

8.  $150^\circ$

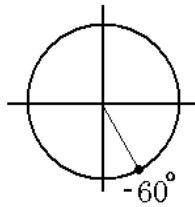
## Answers - Practice Problems

1.)  $180^\circ$ 

$$\cos 180^\circ = -1$$

$$\sin 180^\circ = 0$$

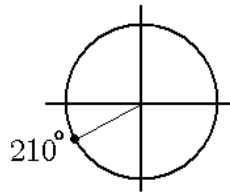
$$\tan 180^\circ = 0$$

2.)  $-60^\circ$ 

$$\cos(-60^\circ) = \frac{1}{2}$$

$$\sin(-60^\circ) = -\frac{\sqrt{3}}{2}$$

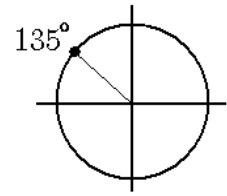
$$\tan(-60^\circ) = -\sqrt{3}$$

3.)  $210^\circ$ 

$$\cos 210^\circ = -\frac{\sqrt{3}}{2}$$

$$\sin 210^\circ = -\frac{1}{2}$$

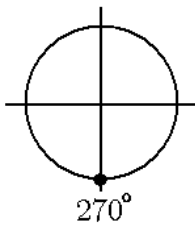
$$\tan 210^\circ = \frac{\sqrt{3}}{3}$$

4.)  $135^\circ$ 

$$\cos 135^\circ = -\frac{\sqrt{2}}{2}$$

$$\sin 135^\circ = \frac{\sqrt{2}}{2}$$

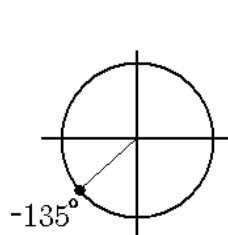
$$\tan 135^\circ = -1$$

5.)  $270^\circ$ 

$$\cos 270^\circ = 0$$

$$\sin 270^\circ = -1$$

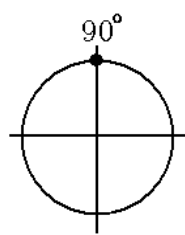
$$\tan 270^\circ = \text{undefined}$$

6.)  $-135^\circ$ 

$$\cos(-135^\circ) = -\frac{\sqrt{2}}{2}$$

$$\sin(-135^\circ) = -\frac{\sqrt{2}}{2}$$

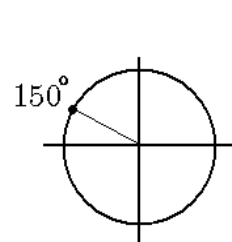
$$\tan(-135^\circ) = 1$$

7.)  $90^\circ$ 

$$\cos 90^\circ = 0$$

$$\sin 90^\circ = 1$$

$$\tan 90^\circ = \text{undefined}$$

8.)  $150^\circ$ 

$$\cos 150^\circ = -\frac{\sqrt{3}}{2}$$

$$\sin 150^\circ = \frac{1}{2}$$

$$\tan 150^\circ = -\frac{\sqrt{3}}{3}$$