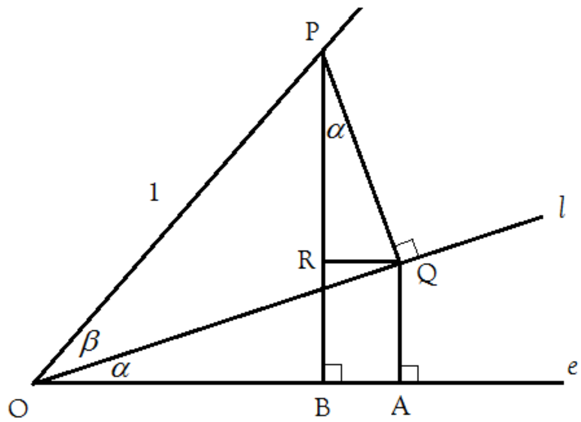
Consider the picture below.



Let O be any point from which we measure the angles α and β as shown. Let P be at a distance of 1 unit to O. We draw two lines passing through P that are perpendicular to rays e and l, respectively. The intersection points created are Q and B. From Q, we draw a line that is perpendicular to BP, the intersection point is R. Finally, we draw a line passing through Q that is perpendicular to ray e. The intersection point is A.

Claim 1: $\angle QPR = \alpha$

proof: First notice that $\angle RQO = \alpha$ because OB and RQ are parallel. Second, $\angle RQO$ and $\angle RQP$ are complementary angles, they add up to 90°.

$$\begin{aligned}\angle RQO + \angle RQP &= 90^\circ \\ \alpha + \angle RQP &= 90^\circ \\ \angle RQP &= 90^\circ - \alpha\end{aligned}$$

Now the angles in triangle PQR must add to 180°:

$$\begin{aligned}\angle QPR + 90^\circ + \angle RQP &= 180^\circ \\ \angle QPR + 90^\circ + 90^\circ - \alpha &= 180^\circ \\ \angle QPR - \alpha &= 0 \\ \angle QPR &= \alpha\end{aligned}$$

Claim 2: $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

proof: Consider the right triangle OPQ.

$$\sin\beta=\frac{PQ}{OP}=\frac{PQ}{1}=PQ \quad\Longrightarrow\quad \boxed{PQ=\sin\beta} \quad \text{and} \quad \cos\beta=\frac{OQ}{OP}=\frac{OQ}{1}=OQ \quad\Longrightarrow\quad \boxed{OQ=\cos\beta}$$

Consider now the right triangle OAQ: $\sin\alpha=\dfrac{AQ}{OQ}=\dfrac{AQ}{\cos\beta} \quad\Longrightarrow\quad AQ=\sin\alpha\cos\beta$

From right triangle PRQ, $\cos\alpha=\dfrac{PR}{PQ}=\dfrac{PR}{\sin\beta} \quad\Longrightarrow\quad PR=\cos\alpha\sin\beta$

From right triangle OBP,

$$\sin(\alpha+\beta)=\frac{BP}{OP}=\frac{BP}{1}=BP=PR+RB=PR+AQ=\cos\alpha\sin\beta+\sin\alpha\cos\beta$$

Claim 3: $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

proof: Consider the right triangle OPQ. $OP=1$ and $PQ=\sin\beta$ and $OQ=\cos\beta$

From right triangle OAQ, $\cos\alpha=\dfrac{AO}{OQ}=\dfrac{AO}{\cos\beta} \quad\Longrightarrow\quad AO=\cos\alpha\cos\beta$

From right triangle PQR, $\sin\alpha=\dfrac{QR}{PQ}=\dfrac{QR}{\sin\beta} \quad\Longrightarrow\quad QR=\sin\alpha\sin\beta$

From right triangle OBP,

$$\cos(\alpha+\beta)=\frac{OB}{OP}=\frac{AO-AB}{1}=AO-QR=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

