Sample Problems

Find the exact value of each of the following and simplify.

1. $2\sin 60° - 5\tan 30°\cot 45°$
2. $2\sin 60° - 3\cos 45° + \tan 30°$
3. $4\cos 30° - \sin 45° + \tan 60°$
4. $\sin 30°\cos 45°\tan 60° - 2\sec 60° + 3\csc 45°\tan 30°\cos 60°$
5. $\dfrac{\sin 60° - 1}{\cos 30° + 1}$

Practice Problems

Find the exact value of each of the following and simplify.

1. $\sin 45° - 2\cos 60°\tan 30° - \csc 45°$
2. $3\tan 60° - 3\cos 45°\sec 60° - \tan 30°\tan 60°\csc 45°$
3. $\dfrac{\cos 45° - \cot 45°}{\sin 45° + \tan 45°}$
4. $\sin^2 30° - \cos 45°\tan^2 60° - 2\cot 60°\tan 60°$
5. $3\sec 30° - \cos 60°\sin 60°$
6. $\dfrac{\tan 60° - \tan 30°}{\tan 60° + \tan 30°}$
7. $\dfrac{\sin 30°\cos 30°\tan 30°\cot 30°}{\sin 45°\cos 45°\tan 45°\cot 45°}$
8. $\dfrac{2\tan 30°}{1 - \tan^2 30°}$
9. $\dfrac{(\sin 30°)(\cos 30°) - (\tan 30°)\cot 30°}{(\sin 45°)(\cos 45°) - (\tan 45°)\cot 45°}$

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Sample Problems - Answers

1.) $-\frac{2\sqrt{3}}{3}$ 2.) $\frac{8\sqrt{3}-9\sqrt{2}}{6}$ or $\frac{4\sqrt{3}}{3}-\frac{3\sqrt{2}}{2}$ 3.) $3\sqrt{3}-\frac{\sqrt{2}}{2}$ or $\frac{6\sqrt{3}-\sqrt{2}}{2}$

4.) $\frac{3}{4}\sqrt{6}-4$ or $\frac{3\sqrt{6}-16}{4}$ 5.) $-7+4\sqrt{3}$

Practice Problems - Answers

1.) $-\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{3}$ 2.) $3\sqrt{3}-4\sqrt{2}$ 3.) $2\sqrt{2}-3$ 4.) $-\frac{3\sqrt{2}}{2}-\frac{7}{4}$ 5.) $\frac{7}{4}\sqrt{3}$ 6.) $\frac{1}{2}$ 7.) $\frac{\sqrt{3}}{2}$

8.) $\sqrt{3}$ 9.) $2-\frac{\sqrt{3}}{2}$

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Sample Problems - Solutions

Find the exact value of each of the following and simplify.

1. $2\sin 60^\circ - 5\tan 30^\circ \cot 45^\circ = -\dfrac{2\sqrt{3}}{3}$

   Solution: We first obtain the exact values of the trigonometric expressions. Using the famous right triangles, we obtain that

   $$\sin 60^\circ = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \frac{1}{\sqrt{3}} \qquad \text{and} \qquad \cot 45^\circ = 1$$

   Because we will be adding and subtracting these expressions, it is best if we rationalize them first.

   $$\tan 30^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

   $$2\sin 60^\circ - 5\tan 30^\circ \cot 45^\circ = 2 \cdot \frac{\sqrt{3}}{2} - 5 \cdot \frac{\sqrt{3}}{3} \cdot 1 = \sqrt{3} - \frac{5\sqrt{3}}{3}$$

   We will then bring these to the common denominator

   $$\sqrt{3} - \frac{5\sqrt{3}}{3} = \frac{3\sqrt{3}}{3} - \frac{5\sqrt{3}}{3} = \frac{3\sqrt{3} - 5\sqrt{3}}{3} = \boxed{\frac{-2\sqrt{3}}{3}}$$

2. $2\sin 60^\circ - 3\cos 45^\circ + \tan 30^\circ$

   Solution: We first obtain the exact values of the trigonometric expressions. Using the famous right triangles, we obtain that

   $$\sin 60^\circ = \frac{\sqrt{3}}{2} \qquad \cos 45^\circ = \frac{1}{\sqrt{2}} \qquad \text{and} \qquad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

   Because we will be adding and subtracting these expressions, it is best if we rationalize them first.

   $$\cos 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad \text{and} \quad \tan 30^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

   $$2\sin 60^\circ - 3\cos 45^\circ + \tan 30^\circ = 2 \cdot \frac{\sqrt{3}}{2} - 3 \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{3} = \sqrt{3} - \frac{3\sqrt{2}}{2} + \frac{\sqrt{3}}{3}$$

   We will then bring these to the common denominator, which is 6 and if there are like terms, we combine them.

   $$\begin{aligned} \sqrt{3} - \frac{3\sqrt{2}}{2} + \frac{\sqrt{3}}{3} &= \frac{6\sqrt{3}}{6} - \frac{9\sqrt{2}}{6} + \frac{2\sqrt{3}}{6} \\ &= \frac{6\sqrt{3} - 9\sqrt{2} + 2\sqrt{3}}{6} = \boxed{\frac{8\sqrt{3} - 9\sqrt{2}}{6}} \end{aligned}$$

   Note: $\dfrac{8\sqrt{3} - 9\sqrt{2}}{6} = \dfrac{8\sqrt{3}}{6} - \dfrac{9\sqrt{2}}{6} = \boxed{\dfrac{4\sqrt{3}}{3} - \dfrac{3\sqrt{2}}{2}}$ is also perfectly acceptable as final answer.

3. $4\cos 30^\circ - \sin 45^\circ + \tan 60^\circ$

   Solution: We first obtain the exact values of the trigonometric expressions. Using the famous right triangles, we obtain that

   $$\cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \sin 45^\circ = \frac{1}{\sqrt{2}} \qquad \text{and} \qquad \tan 60^\circ = \sqrt{3}$$

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Because we will be adding and subtracting these expressions, it is best if we rationalize them first.

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$4\cos 30^\circ - \sin 45^\circ + \tan 60^\circ = 4 \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} + \sqrt{3} = 2\sqrt{3} - \frac{\sqrt{2}}{2} + \sqrt{3} = \boxed{3\sqrt{3} - \frac{\sqrt{2}}{2}}$$

Note: We can also bring these to the common denominator.

$$3\sqrt{3} - \frac{\sqrt{2}}{2} = \frac{6\sqrt{3}}{2} - \frac{\sqrt{2}}{2} = \boxed{\frac{6\sqrt{3} - \sqrt{2}}{2}}$$

is also perfectly acceptable.

4. $\sin 30^\circ \cos 45^\circ \tan 60^\circ - 2\sec 60^\circ + 3\csc 45^\circ \tan 30^\circ \cos 60^\circ = \frac{3}{4}\sqrt{6} - 4$

Solution: We first obtain the exact values of the trigonometric expressions. Using the famous right triangles, we obtain that

$$\sin 30^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \tan 60^\circ = \sqrt{3} \quad \sec 60^\circ = 2 \quad \csc 45^\circ = \sqrt{2} \quad \tan 30^\circ = \frac{1}{\sqrt{3}} \quad \text{and} \quad \cos 60^\circ = \frac{1}{2}$$

Because we will be adding and subtracting these expressions, it is best if we rationalize them first.

$$\cos 45^\circ = \frac{\sqrt{2}}{2} \quad \text{and} \quad \tan 30^\circ = \frac{\sqrt{3}}{3}$$

$$\sin 30^\circ \cos 45^\circ \tan 60^\circ - 2\sec 60^\circ + 3\csc 45^\circ \tan 30^\circ \cos 60^\circ =$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \cdot \sqrt{3} - 2 \cdot 2 + 3 \cdot \sqrt{2} \cdot \frac{\sqrt{3}}{3} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} - 4 + \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{4} - 4 + \frac{\sqrt{6}}{2} = \frac{\sqrt{6}}{4} - 4 + \frac{2\sqrt{6}}{4} = \boxed{\frac{3\sqrt{6}}{4} - 4}$$

Note: We can also bring these to the common denominator.

$$\frac{3\sqrt{6}}{4} - 4 = \frac{3\sqrt{6}}{4} - \frac{16}{4} = \boxed{\frac{3\sqrt{6} - 16}{4}}$$

is also perfectly acceptable.

5. $\dfrac{\sin 60^\circ - 1}{\cos 30^\circ + 1} = -7 + 4\sqrt{3}$

Solution: We first obtain the exact values of the trigonometric expressions. Using the famous right triangles, we obtain that

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \text{and} \quad \cos 30^\circ = \frac{\sqrt{3}}{2} \text{ and so } \frac{\sin 60^\circ - 1}{\cos 30^\circ + 1} = \frac{\frac{\sqrt{3}}{2} - 1}{\frac{\sqrt{3}}{2} + 1}$$

To clear the denominators, we multiply both numerator and denominator by 2

$$\frac{\frac{\sqrt{3}}{2} - 1}{\frac{\sqrt{3}}{2} + 1} = \frac{2\left(\frac{\sqrt{3}}{2} - 1\right)}{2\left(\frac{\sqrt{3}}{2} + 1\right)} = \frac{\sqrt{3} - 2}{\sqrt{3} + 2}$$

We rationalize our result:

$$\frac{\sqrt{3} - 2}{\sqrt{3} + 2} = \frac{\sqrt{3} - 2}{\sqrt{3} + 2} \cdot \frac{\sqrt{3} - 2}{\sqrt{3} - 2} = \frac{(\sqrt{3} - 2)^2}{3 - 4} = \frac{3 - 4\sqrt{3} + 4}{-1} = \frac{7 - 4\sqrt{3}}{-1} = -\left(7 - 4\sqrt{3}\right) = \boxed{-7 + 4\sqrt{3}}$$

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